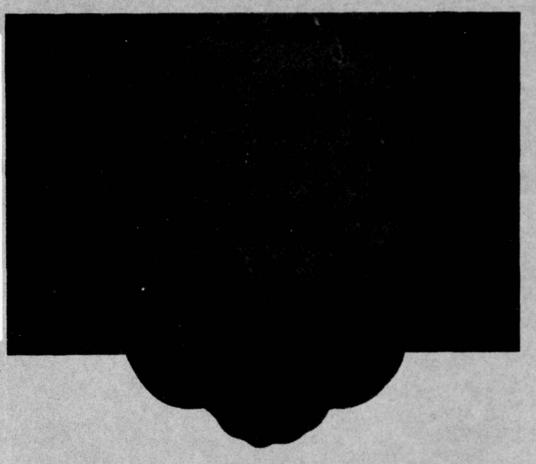


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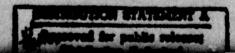
THE COVARIANCE STRUCTURE OF THE DEPARTURE PROCESS:
GI/M/1 and M/G/1 Queues

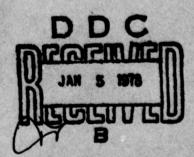
Nancy Gustafson

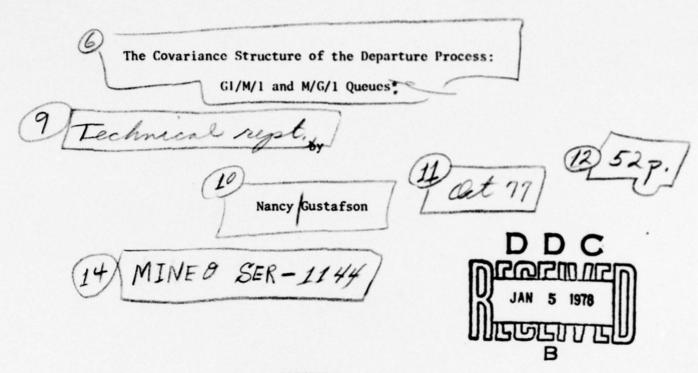
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ABSTRACT

Formulae, suitable for computation, for the lag n autocorrelation of departure intervals in the GI/M/1 queues are developed for n = 1 to 6. A generating function, comparable to the one known for the M/G/1 case, is found for GI/M/1. For the E_k /M/1 and M/ E_k /1 queues, expressions for arbitrary lag are given. Tables and the APL functions used to produce them are provided for E_k /M/1, H/M/1, and M/ E_k /1.

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1. Introduction

Knowledge of the departure process of a queue is essential to the analysis of networks in which the output of one queue forms the input to another. Of particular interest is the covariance structure of the departures. The work in this area to date is summarized by Reynolds (1975) and Daley (1976).

The papers serving as points of departure for this report are those of Jenkins (1966) and Daley (1968). In the former, the lag one and two autocorrelations of departure intervals are derived for queues with independent Erlangian service times and Poisson arrivals. His method may be extended to general service time distributions, but only with considerable difficulty to lags of greater than two. Daley (1968) provides a generating function for the autocovariances of M/G/1 departure intervals. He also gives a rather difficult expression for the lag n covariance in the GI/M/1 case, suggesting that it can be used to derive a generating function. This derivation, the result of which has a comparatively simple form, is carried out in section 3 of this paper.

The main purpose here is to convert the GI/M/I and M/G/I generating functions for the departure interval covariances to explicit formulae which may be used directly to obtain results for special cases. Tables of autocorrelations are given for $E_k/M/I$, H/M/I, and $M/E_k/I$ queues with various traffic intensities. The special Erlang and hypergeometric distributions were chosen because they are so frequently suggested as approximations to actual arrival and/or service distributions. To match a coefficient of variation less than one, an Erlang is used;

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otherwise, the hypergeometric.

2. Notation

The queue assumed throughout possesses a single server and infinite waiting room. Customers arrive according to a renewal process and are served in order of arrival. Service times are i.i.d. and independent of the arrival process. Notation is as follows:

 T_n = interval between arrivals of customers n and n + 1

 W_n = time spent in queue (exclusive of service) by customer n

 $S_n =$ service time for customer n

 D_n = interval between departures of customers n - 1 and n

Arrival intervals and service times have distribution functions A(x) and B(x), respectively, with associated Laplace-Stieltjes transforms

$$A^*(s) = E[e^{-sT}]$$
 and $B^*(s) = E[e^{-sS}]$.

The rate of arrivals is λ , the service rate is μ , and the traffic intensity λ/μ is denoted by ρ .

 $\mathbf{E}_{\mathbf{k}}$ stands for the (special) Erlang distribution with k phases, p.d.f.

$$f(t) = \frac{(k\lambda)^k t^{k-1} e^{-k\lambda t}}{(k-1)!}, t \ge 0.$$

H stands for the hypergeometric distribution with p.d.f.

$$f(t) = 2q^{2}\lambda e^{-2q\lambda t} + 2(1-q)^{2}\lambda e^{-2(1-q)\lambda t}, \quad t \ge 0,$$

3. Autocorrelations of Departure Intervals: GI/M/1.

Daley (1968) gives as his Theorem 5 the result that the departure intervals in a stationary GI/M/1 queue have

(3.1)
$$\operatorname{cov}(D_{0}, D_{n}) = (\frac{1}{\rho} - \frac{1}{s_{0}})(\frac{1}{\mu})[E(D_{n}|W_{0} = 0) - E(D_{0})],$$

where s is the root of smallest absolute value of

$$s = A*(\mu[1 - s]),$$

or alternatively,

(3.2)
$$\operatorname{cov}(D_0, D_n) = (\frac{1}{\rho} - \frac{1}{s_0})(\frac{1}{\mu})E(W_n - W_{n-1}|W_0 = 0).$$

A method to find the expectation in (3.2) is given in Takaćs (1962). It is simpler, however, to use Takaćs' generating function for $E(W_n|W_0=0)$ to derive a generating function for the autocovariances, then use the result to obtain explicit expressions for $cov(D_0,D_n)$.

Making certain changes in equation (28) on page 121 of Takaćs (customer n here corresponds to his customer n+1), one obtains

(3.3)
$$\sum_{n=1}^{\infty} E[e^{-sW_{n-1}}|W_{0} = 0]z^{n} = \frac{z[1-s_{1}(z)]}{(1-z)[1-\frac{\mu s_{1}(z)}{\mu+s}]},$$

where s₁(z) is the root of smallest absolute value of

$$s = z A^* (\mu[1 - s])$$
.

Evaluating at s = 0 the partial derivative of each side with respect to s results in

$$\sum_{n=1}^{\infty} E(W_{n-1} | W_0 = 0) z_n = \frac{z s_1(z)}{\mu(1-z)[1-s_1(z)]},$$

and

$$\sum_{n=1}^{\infty} E(W_n | W_0 = 0) z^n = \sum_{n=0}^{\infty} E(W_n | W_0 = 0) z^n$$

$$= \sum_{n=1}^{\infty} E(W_{n-1} | W_0 = 0) z^{n-1}$$

$$= \frac{s_1(z)}{\mu(1-z)[1-s_1(z)]}.$$

Combining these yields

(3.4)
$$\sum_{n=1}^{\infty} E(W_n - W_{n-1} | W_0 = 0) z^n = \frac{s_1(z)}{\mu[1 - s_1(z)]}, |z| < 1.$$

Substituting in (3.2), the desired generating function is found to be

(3.5)
$$\sum_{n=1}^{\infty} \operatorname{cov}(D_{0}, D_{n}) z^{n} = (\frac{1}{\rho} - \frac{1}{s_{0}}) (\frac{1}{\mu^{2}}) \{ \frac{s_{1}(z)}{1 - s_{1}(z)} \},$$

where s_0 and $s_1(z)$ are the roots of smallest absolute value of $s = A*(\mu[1 - s])$ and $s = zA*(\mu[1 - s])$, respectively.

To determine the autocovariance lag n for specific n, it is first necessary to expand $s_1(z)$ in z. Takaćs (Lemma 1, page 113) provides the formula

(3.6)
$$s_1(z) = z \sum_{j=1}^{\infty} \frac{(-\mu z)^{j-1}}{j!} \left[\frac{d^{j-1} \{A^*(s)\}^j}{d s^{j-1}} \right]_{s=\mu}.$$

For compactness of notation, let $\alpha_j = \frac{d^j A^*(s)}{d s^j}\Big|_{s=\mu}$.

$$\begin{split} s_1(z) &= z\alpha_0 - z^2\mu\alpha_0\alpha_1 + \frac{z^3\mu^2}{2!} \left\{ 2\alpha_0\alpha_1^2 + \alpha_0^2\alpha_2 \right\} \\ &- \frac{z^4\mu^3}{3!} \left\{ 6\alpha_0\alpha_1^3 + 9\alpha_0^2\alpha_1\alpha_2 + \alpha_0^3\alpha_3 \right\} \\ &+ \frac{z^5\mu^4}{4!} \left\{ 24\alpha_0\alpha_1^4 + 72\alpha_0^2\alpha_1^2\alpha_2 + 12\alpha_0^3\alpha_2^2 + 16\alpha_0^3\alpha_1\alpha_3 + \alpha_0^4\alpha_4 \right\} \\ &- \dots ; \\ \left[s_1(z) \right]^2 &= z^2\alpha_0^2 - 2z^3\mu\alpha_0^2\alpha_1 + z^4\mu^2 \left\{ 3\alpha_0^2\alpha_1^2 + \alpha_0^3\alpha_2 \right\} \dots ; \\ \left[s_1(z) \right]^3 &= z^3\alpha_0^3 - 3z^4\mu\alpha_0^3\alpha_1 + 2^5\mu^2 \left\{ 6\alpha_0^3\alpha_1^2 + \frac{3}{2}\alpha_0^4\alpha_2 \right\} \dots ; \end{split}$$

and so on.

Carrying out these expansions to a number of terms, working out the first six terms of

$$\frac{s_1(z)}{1-s_1(z)} = \sum_{j=1}^{\infty} [s_1(z)]^j ,$$

and multiplying by $(\frac{1}{\rho} - \frac{1}{s_0})(\frac{1}{\mu^2})$, one obtains the desired autocovariances. From Daley (1968),

(3.7)
$$\operatorname{Var}(D_0) = \operatorname{Var}(T_0) - (\frac{1}{\rho} - \frac{1}{s_0}) \frac{2s_0}{\mu^2(1-s_0)}.$$

The autocorrelations, lags 1 through 6, of departure intervals for the GI/M/1 queue are as follows:

$$(3.8) \qquad \operatorname{corr}(D_0, D_1) = M\alpha_0$$

$$\operatorname{corr}(D_0, D_2) = M[\alpha_0^2 - \mu\alpha_0\alpha_1]$$

$$\operatorname{corr}(D_0, D_3) = M[\alpha_0^3 - 2\mu\alpha_0^2\alpha_1 + \frac{\mu^2}{2!} (2\alpha_0\alpha_1^2 + \alpha_0^2\alpha_2)]$$

$$\operatorname{corr}(D_0, D_4) = M[\alpha_0^4 - 3\mu\alpha_0^3\alpha_1 + \frac{\mu^2}{2!} (6\alpha_0^2\alpha_1^2 + 2\alpha_0^3\alpha_2)$$

$$- \frac{\mu^3}{3!} (6\alpha_0\alpha_1^3 + 9\alpha_0^2\alpha_1\alpha_2 + \alpha_0^3\alpha_3)]$$

$$\operatorname{corr}(D_0, D_5) = M[\alpha_0^5 - 4\mu\alpha_0^4\alpha_1 + \frac{\mu^2}{2!} (12\alpha_0^3\alpha_1^2 + 3\alpha_0^4\alpha_2)$$

$$- \frac{\mu^3}{3!} (24\alpha_0^2\alpha_1^3 + 24\alpha_0^3\alpha_1\alpha_2 + 2\alpha_0^4\alpha_3)$$

$$+ \frac{\mu^4}{4!} (24\alpha_0\alpha_1^4 + 72\alpha_0^2\alpha_1^2\alpha_2 + 12\alpha_0^3\alpha_2^2 + 16\alpha_0^3\alpha_1\alpha_3 + \alpha_0^4\alpha_4)]$$

$$\operatorname{corr}(D_0, D_6) = M[\alpha_0^6 - 5\mu\alpha_0^5\alpha_1 + \frac{\mu^2}{2!} (20\alpha_0^4\alpha_1^2 + 4\alpha_0^5\alpha_2)$$

$$- \frac{\mu^3}{3!} (60\alpha_0^3\alpha_1^3 + 45\alpha_0^4\alpha_1\alpha_2 + 3\alpha_0^5\alpha_3)$$

$$+ \frac{\mu^4}{4!} (120\alpha_0^2\alpha_1^4 + 240\alpha_0^3\alpha_1^2\alpha_2 + 30\alpha_0^4\alpha_2^2 + 40\alpha_0^4\alpha_1\alpha_3 + 2\alpha_0^5\alpha_4)$$

$$- \frac{\mu^5}{5!} (120\alpha_0\alpha_1^5 + 600\alpha_0^2\alpha_1^3\alpha_2 + 300\alpha_0^3\alpha_1\alpha_2^2 + 200\alpha_0^3\alpha_1^2\alpha_3$$

$$+ 50\alpha_0^4\alpha_2\alpha_3 + 25\alpha_0^4\alpha_1\alpha_4 + \alpha_0^5\alpha_5)$$

where M = $\frac{1}{\mu^2} \left[\text{Var T}_0 \left(\frac{1}{\rho} - \frac{1}{s_0} \right)^{-1} - \frac{2s_0}{\mu^2 (1-s_0)} \right]^{-1}$, with s_0 and α_j defined as above.

There apparently exists an expression for lag of arbitrary n, starting

(3.9)
$$\operatorname{corr}(D_0, D_n) = M\{\alpha_0^n - (n-1)\mu\alpha_0^{n-1}\alpha_1 + \frac{\mu^2}{2!} \left[\frac{(n-1)!}{(n-3)!} \alpha_0^{n-2}\alpha_1^2 + (n-2)\alpha_0^{n-1}\alpha_2 \right] - \frac{\mu^3}{3!} \left[\frac{(n-1)!}{(n-4)!} \alpha_0^{n-3}\alpha_1^3 + 3(n-1)(n-3)\alpha_0^{n-2}\alpha_1\alpha_2 + (n-3)\alpha_0^{n-1}\alpha_3 \right] + \dots \},$$

but its complete statement requires further work.

For the Ek/M/1 case,

$$\alpha_{j} = \frac{(k+n-1)!}{(k-1)!} \left(\frac{-1}{k\rho\mu}\right)^{n} \left(\frac{k\rho}{k\rho+1}\right)^{k+j}$$
,

and inspection of the results of substituting this in (3.8) reveals a general formula:

(3.10)
$$\operatorname{corr}(D_0, D_n) = M(\frac{k\rho}{k+1})^{nk} \sum_{i=0}^{n-1} {nk+i \choose i} \frac{k(n-i)}{(nk+i)(\rho k)^i} (\frac{k\rho}{k\rho+1})^i$$
,

where
$$M = \left[\frac{1}{k\rho^2} \left(\frac{1}{\rho} - \frac{1}{s_0}\right)^{-1} - \frac{2s_0}{1-s_0}\right]^{-1}$$
.

Tables for the autocorrelations of lags one through five for $E_k/M/1$ queues with various k and ρ , and for H/M/1 queues with various q and ρ , are given in section 5 (Table I). The last column of these tables, labelled "TOTAL", contains the values of

(3.11)
$$\sum_{n=1}^{\infty} \operatorname{corr}(D_0, D_n) = M(\frac{s_0}{1-s_0}).$$

"CV" denotes the coefficient of variation of T_0 for the given k or q.

To find M requires determination of the root s_0 . The method of "false position", which was recommended for this problem by Sahin (1970), proved quite fast, particularly when used with Aitken's method of acceleration. The algorithm and computer routine used are described in Appendix 1. The other APL functions required are displayed and discussed briefly in Appendix 2.

4. Autocorrelations of Departure Intervals: M/G/1.

For the M/G/1 case, Daley (1968) provides the generating function:

(4.1)
$$\sum_{n=1}^{\infty} \operatorname{cov}(D_0, D_n) z^n = (\frac{1-\rho}{\lambda^2}) \{ \frac{s_1(z)-z}{(1-z)[1-s_1(z)]} + \frac{zs_1'(z)-s_1(z)}{s_1(z)s_1'(z)(1-z)} \} |z| < 1,$$

where s₁(z) is the root of smallest absolute value of

$$s_1 = zB^*(\lambda[1 - s]),$$

and

$$s_1'(z) = \frac{d s_1'(x)}{dx} \Big|_{x=z}$$
.

The expansion of $s_1(z)$ is the same as that given in section 3, with $\beta_j = \frac{d^j B^\star(s)}{d \ s^j} \bigg|_{s=\lambda} \text{ replacing } \alpha_j \text{ . Also needed is}$

$$s_1'(z) = \beta_0 - 2z\lambda\beta_0\beta_1 + \frac{3z^2\lambda^2}{2!} \left[2\beta_0\beta_1^2 + \beta_0^2\beta_2\right] - \frac{4z^3\lambda^3}{3!} \left[6\beta_0\beta_1^3 + 9\beta_0^2\beta_1\beta_2 + \beta_0^3\beta_3\right] + \dots$$

Note that s_0 is not involved here. However, in other aspects the M/G/1 case is computationally more difficult than the GI/M/1. The expression in brackets in (4.1) can be rewritten as

$$(4.2) (1+z+z^2+\ldots)\{[s_1(z)-z][1+s_1(z)+s_1^2(z)+\ldots]+\frac{z}{s_1(z)}-\frac{1}{s_1(z)}\}.$$

By methods similar to those of section 3 one obtains

$$[s_{1}(z) - z][1 + s_{1}(z) + s_{1}^{2}(z) + \dots] = z[-1 + \beta_{0}] - z^{2}[\lambda\beta_{0}\beta_{1} + \beta_{0} - \beta_{0}^{2}]$$

$$+ z^{3}[\lambda^{2}(\beta_{0}\beta_{1}^{2} + \frac{1}{2}\beta_{0}^{2}\beta_{2}) + \lambda(\beta_{0}\beta_{1} - 2\beta_{0}^{2}\beta_{1}) - \beta_{0}^{2} + \beta_{0}^{3}]$$

$$- z^{4}[\lambda^{3}(\beta_{0}\beta_{1}^{3} + \frac{3}{2}\beta_{0}^{2}\beta_{1}\beta_{2} + \frac{1}{6}\beta_{0}^{3}\beta_{3})$$

$$+ \lambda^{2}(\beta_{0}\beta_{1}^{2} + \frac{1}{2}\beta_{0}^{2}\beta_{2} - 3\beta_{0}^{2}\beta_{1}^{2} - \beta_{0}^{3}\beta_{2})$$

$$+ \lambda(-2\beta_{0}^{2}\beta_{1} + 3\beta_{0}^{3}\beta_{1}) + \beta_{0}^{3} - \beta_{0}^{4}] + \dots$$

The remaining two terms in (4.2) must be expanded in Taylor series about zero $(s_1(z) \text{ and } s_1'(z) \text{ are non-vanishing in } |z| < 1)$. The results of this tedious procedure are

(4.4)
$$\frac{z}{s_1(z)} = \frac{1}{\beta_0} + z\lambda \left(\frac{\beta_1}{\beta_0}\right) - \frac{z^2\lambda^2}{2!} (\beta_2) + \frac{z^3\lambda^3}{3!} (3\beta_1\beta_2 + \beta_0\beta_3) - \frac{z^4\lambda^4}{4!} (12\beta_1^2\beta_2 + 6\beta_0\beta_2^2 + 8\beta_0\beta_1\beta_3 + \beta_0^2\beta_4) + \dots$$

and

(4.5)
$$\frac{1}{s_1'(z)} = \frac{1}{\beta_0} + z\lambda \left(\frac{2\beta_1}{\beta_0}\right) - \frac{z^2\lambda^2}{2!} \left(3\beta_2 - \frac{2\beta_1^2}{\beta_0}\right) + \frac{z^3\lambda^3}{3!} \left(4\beta_0\beta_3\right) - \frac{z^4\lambda^4}{4!} \left(6\beta_0\beta_2^2 + 16\beta_0\beta_1\beta_3 + 5\beta_0^2\beta_4\right) + \dots$$

Subtracting (4.5) from (4.4), adding (4.3), then multiplying by $\sum_{j=0}^{\infty} z^j$, one finally has an expansion for (4.2). Using this and

(4.6)
$$Var(D_0) = Var(S_0) + (\frac{1-\rho^2}{\lambda^2})$$
,

the autocorrelations for the M/G/1 queue are found to be

$$(4.7) \quad \operatorname{corr}(D_0, D_1) = M[-1 + \beta_0 - \frac{\lambda \beta_1}{\beta_0}]$$

$$\operatorname{corr}(D_0, D_2) = M[-1 + \beta_0^2 - \lambda (\frac{\beta_1}{\beta_0} + \beta_0 \beta_1) + \lambda^2 (\beta^2 - \frac{\beta_1^2}{\beta_0})]$$

$$\operatorname{corr}(D_0, D_3) = M[-1 + \beta_0^3 - \lambda (\frac{\beta_1}{\beta_0} + 2\beta_0^2 \beta_1) + \lambda^2 (\beta_2 - \frac{\beta_1^2}{\beta_0} + \beta_0 \beta_1^2 + \frac{\beta_0^2 \beta_2}{2})$$

$$- \lambda^3 (\frac{-\beta_1 \beta_2}{2} + \frac{\beta_0 \beta_3}{2})]$$

$$\operatorname{corr}(D_0, D_4) = M[-1 + \beta_0^4 - \lambda (\frac{\beta_1}{\beta_0} + \beta_0^3 \beta_1) + \lambda^2 (\beta_2^2 - \frac{\beta_1^2}{\beta_0} + 3\beta_0^2 \beta_1^2 + \beta_0^3 \beta_2)$$

$$- \lambda^3 (\frac{-\beta_1 \beta_2}{2} + \frac{\beta_0 \beta_3}{2} + \beta_0 \beta_1^3 + \frac{3\beta_0^2 \beta_1 \beta_2}{2} + \frac{\beta_0^3 \beta_3}{6})$$

$$+ \lambda^4 (\frac{-\beta_1^2 \beta_2}{2} + \frac{\beta_0 \beta_1 \beta_3}{3} + \frac{\beta_0^2 \beta_4}{6})]$$

$$\operatorname{corr}(D_0, D_5) = M[-1 + \beta_0^5 - \lambda (\frac{\beta_1}{\beta_0} + 4\beta_0^4 \beta_1) + \lambda^2 (\beta_2 - \frac{\beta_1^2}{\beta_0} + 6\beta_0^3 \beta_1^2 + \frac{3\beta_0^4 \beta_2}{2})$$

$$- \lambda^3 [\frac{-\beta_1 \beta_2}{2} + \frac{\beta_0 \beta_1 \beta_3}{3} + 4\beta_0^2 \beta_1^3 + 4\beta_0^3 \beta_1 \beta_2 + \frac{\beta_0^4 \beta_3}{3})$$

$$+ \lambda^4 (\frac{-\beta_1^2 \beta_2}{2} + \frac{\beta_0 \beta_1 \beta_3}{3} + \frac{\beta_0^2 \beta_4}{6} + \beta_0 \beta_1^4 + 3\beta_0^2 \beta_1^2 \beta_2$$

$$+ \frac{\beta_0^3 \beta_2}{2} + \frac{2\beta_0^3 \beta_1 \beta_3}{3} + \frac{\beta_0^4 \beta_4}{24})$$

$$- \lambda^5 (\frac{-\beta_1^3 \beta_2}{2} - \frac{\beta_0 \beta_1 \beta_2^2}{4} + \frac{\beta_0 \beta_1^2 \beta_3}{6} + \frac{\beta_0^2 \beta_2 \beta_3}{6} + \frac{\beta_0^2 \beta_1 \beta_4}{24} + \frac{\beta_0^3 \beta_5}{24})]$$

$$\begin{aligned} & \operatorname{corr}(D_0,D_6) = M[-1 + \beta_0^6 - \lambda(\frac{\beta_1}{\beta_0} + 5\beta_0^5\beta_1) + \lambda^2(\beta_2 - \frac{\beta_1^2}{\beta_0} + 10\beta_0^4\beta_1^2) \\ & + 2\beta_0^5\beta_2) - \lambda^3(\frac{-\beta_1\beta_2}{2} + \frac{\beta_0\beta_3}{2} + 10\beta_0^3\beta_1^3 + \frac{15\beta_0^4\beta_1\beta_2}{2} + \frac{\beta_0^5\beta_3}{2}) \\ & + \lambda^4(\frac{-\beta_1^2\beta_2}{2} + \frac{\beta_0\beta_1\beta_3}{3} + \frac{\beta_0^2\beta_4}{6} + 5\beta_0^2\beta_1^4 + 10\beta_0^3\beta_1^2\beta_2) \\ & + \frac{5\beta_0^4\beta_2^2}{4} + \frac{5\beta_0^4\beta_1\beta_3}{3} + \frac{\beta_0^5\beta_4}{12} \\ & - \lambda^5(\frac{-\beta_1^3\beta_2}{2} - \frac{\beta_0\beta_1\beta_2^2}{4} + \frac{\beta_0\beta_1^2\beta_3}{6} + \frac{\beta_0\beta_2\beta_3}{6} + \frac{\beta_0\beta_2\beta_3}{4} + \frac{7\beta_0^2\beta_1\beta_4}{24} \\ & + \frac{\beta_0^3\beta_5}{24} + \beta_0\beta_1^5 + 5\beta_0^2\beta_1^3\beta_2 + \frac{5\beta_0^3\beta_1\beta_2^2}{2} + \frac{5\beta_0^3\beta_1\beta_3}{3} \\ & + \frac{5\beta_0^4\beta_2\beta_3}{12} + \frac{5\beta_0^4\beta_1\beta_4}{24} + \frac{\beta_0^5\beta_5}{120}) + \lambda^6(\frac{-\beta_1^4\beta_2}{2} - \frac{3\beta_0\beta_1^2\beta_2^2}{4} \\ & + \frac{\beta_0^2\beta_1\beta_2\beta_3}{2} + \frac{3\beta_0^2\beta_1^2\beta_4}{8} + \frac{\beta_0^3\beta_3^2}{12} + \frac{\beta_0^3\beta_2\beta_4}{6} + \frac{7\beta_0^3\beta_1\beta_5}{60} + \frac{\beta_0^4\beta_6}{120})], \end{aligned}$$

where

$$M = \left[\frac{\lambda^2 \, \text{Var}(S_0)}{1-\rho} + \rho + 1\right]^{-1} ,$$

with β_i as defined above.

As in the $\rm E_k/M/1$ case, the expressions for $\rm M/E_k/1$ autocorrelations are considerably simpler, and may be generalized to

$$(4.8) \quad \operatorname{corr}(D_0, D_n) = M[-1 + \sum_{i=0}^{n-1} {i \cdot (k+1) + 1 \choose i} (\frac{k}{i \cdot (k+1) + 1}) (\frac{k}{k})^{i+1} (\frac{k}{k+\rho})^{i \cdot (k+1)} + (\frac{k}{k+\rho})^{nk} \sum_{i=0}^{n-1} {nk+i \choose i} (\frac{k}{nk+i})^{i} (\frac{k}{k+\rho})^{i}],$$

where

$$M = \frac{k(1-\rho)}{\rho^2(1-k)+k}.$$

This leads to the same formulae as given by Jenkins (1966) for lags one and two.

Tables for autocorrelations lags one through five for various $M/E_{\bf k}/1$ queues are provided in the following section (Table II). The last column of each table gives the sum

(4.9)
$$\sum_{n=1}^{\infty} \operatorname{corr}(D_0, D_n) = M[\frac{\rho^2(k-1)}{2k(1-\rho)}].$$

"CV" is the coefficient of variation of S_0 . APL functions used are described in Appendix 2.

Acknowledgements

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Appendix I. Finding the Root so.

The APL function used here to obtain values for the root so of the equation $s = A*(\mu[1-s])$ is based on a program written by James Kitchen. Modifications were made to take advantage of the specific situation, and to allow for a vector argument. The routine uses the method of "false position", with Aitken's acceleration applied on every third iteration.

Algorithm to find the root of f(x) = 0 between ℓ_0 and u_0

- Choose upper and lower values ℓ_0 and \mathbf{u}_0 which enclose the root.
- Initialize i(the iteration counter) and j(the acceleration flag) to 0.
- 3. Set i = i + 1. 4. Set $x_i = \frac{u_i f(\ell_i) \ell_i f(u_i)}{f(\ell_i) f(u_i)}$, and $y_i = f(x_i)$.
- 5. Set j = j + 1, and $z_j = x_i$.
- 6. If $|y_i| < \text{tolerance}$, stop. If not, go on to (7).
- 7. If $y_i > 0$, let $u_{i+1} = x_i$, $\ell_{i+1} = \ell_i$; if not, let $u_{i+1} = u_i$, $\ell_{i+1} = x_i$.
- 8. If j = 3, do acceleration routine; it not, go to (3).

- Acceleration routine: 1. Let $x = z_3 \frac{(z_3 z_2)^2}{z_1 2z_2 + z_3}$ and y = f(x).
- 2. If $\ell_{i+1} < x < u_{i+1}$, set $x_{i+1} = x$, $y_{i+1} = y$, and j = 0, then go to (6). If not, reject the accelerated result, setting j = 1, $z_1 = z_3$, and returning to (3).

The APL function SØ EROOTS K takes as arguments the number of stages K for the Erlang distribution desired, and an initial guess SØ, a vector of initial upper bounds on s_0 for the n values of ρ being considered. The initial lower bounds

used are ρ^k . When $f(s_0) = s_0 - A^*(1 - s_0) < TOL$ for each ρ , the function returns the vector of roots. Setting $TOL = 5 \times 10^{-7}$, no more than three or four iterations were necessary.

Warning: this function, as written, requires a minimum of 5 values of $\boldsymbol{\rho}.$

VEROOTS[D]V
7 S+SU EROOTS K; IACCEL; XHOLD; X; Y; XT; YT; N; SGN; RHOM
[1] ofinds root for ek/m/1: for each intensity in vector RHO
[2] QUSES METHOD OF FALSE POSITION WITH AITKEN'S ACCELERATION
[3] ARHO AND TOL MUST BE SUPPLIED
[4] XHOLD+((N+pRHO),3)p8.5
[5] RHOM-Q(pQX-Q(2,N)p((RHO*K),S0))pRHO
[6] Y+X-÷(1+(1-X)÷KxRHOM)*K
[7] IACCEL+ITER+0
[0] MAIN:ITER+ITER+1
[9] XT+(-/Y×ΦX)+-/Y
[10] YT+XT-+(1+(1-XT)+K×RHO)+K
[11] XHOLDE; IACCEL-IACCEL+1]-XT
[12] WITHIN: -FINALXITOL>[/ YT
[13] X[;SGN+(YT[1]>0)+1]+XT
[14] Y[;SGN]+YT
[!5] -MAIN×11ACCEL<3
[16] ACCELERATED X NOW TRIED
[! 7] XT+XHOLD[;3]-((XHOLD[;3]-XHOLD[;1])*2)++/((N,3)p 1 72 1)*XHOLD
[18] YT-XT-÷(1+(1-XT)+KXRHO)+K
[19] -WITHINX1~IACCEL+(XT[5] <x[5;1])vxt[5]>X[5;2]</x[5;1])vxt[5]>
[#03 XHOLDE:11+XHOLDE:31
[21] →MAIN
[22] FINAL: S+XT

Appendix II. APL Functions Used

To produce the tables for $E_k/M/1$ and H/M/1 queues, an APL function which could be altered to fit any appropriate arrival distribution was devised.

ECORR takes as arguments the starting vector (R) for the root-finding routine EROOTS described in Appendix I, and a vector of values (KV) for the Erlang parameter k. KV must be arranged in increasing size, and contain at least two values. R is usually set to RHO, the vector containing the various traffic intensities for which the autocorrelations are to be computed. The value of FORMAT used for all the tables is displayed following the functions below.

HCORR takes only one argument, the vector of parameters QV. To obtain the values for QV corresponding to a vector of coefficients of variation, use the function CONVERT. The roots for H/M/1 are simply

$$s_0 = \rho + \frac{(1-\sqrt{1-4\rho(1-\rho)(1-2q)^2})}{2}$$
,

and are provided to HCORR by HROOTS.

To adapt either of these functions for another arrival distribution, make the following substitutions:

- (a) $V \leftarrow \text{expression for } Var(T_0)$
- (b) M + expression for M [see (3.8)]
- (c) to (d) expressions for C1 = α_0 , A1 = α_1 , A2 = α_2 , A3 = α_3 , and A4 = α_4
- (e) Where θ is the parameter to be printed, with n decimal places, the statement should be

$$\Box \leftarrow$$
' $\theta =$ ', $(n \otimes \theta)$ ', $CV =$ ', $3 \otimes (expression for c. of v.)$

(f) + number of statement labelled (a).

The function for $M/E_k/1$ is designed specifically for Erlangian service times. A routine alterable for use with any M/G/1 queue would be possible, but unwieldy. CORRE takes the argument KV, the vector of values for k, and uses the pre-set variables RHO and FORMAT described above.

25	SAV IS THE VECTOR OF VALUES FOR Q: SHO AND FORMAT MUST BE PROVIDED
1	U+(1++0×0×1-6+0V[1])+9±0×2
(b) (s)	X++(Y+(+8380)++5)-2x5+1-5-139 833015 9
7	の 1 T 中の 1 L O
	-
[0]	AI(N1+D1×2)+N2+D2×2
7.8	11.5
	C
	E
	(,)
-	ن
-	S
-]
C	1.0 2.1
	1 (Caf): 1 = 0
-] ' 330 N= 1 - 5 - TOTAL!
1	-
7	7-502/ATPERED. 0((6.N)0N)x(0.N+2016)0C1.C2.C3.C4.C5.CT
7	i
0	
77	7-
	A S SCOR KURKIDITIUM TOTAL CONCORTOR OF THE CONTROL
[]	OTABLE OF 1.46 1.2.3.4.5 AND TOTAL AUTOCORRELATIONS FOR EX.M.
[3]	ANY IS THE VECTOR OF VALUES FOR K: THO AND FORMAT MUST
[3]	AR IS THE START FOR EROOTS: MAY AS RHO IF DESIRED
[4]	
5	
13	1
	ST 1754 LO
15	1
55	
	1
1212	1
613	
[14]	1
5	
[16]	2 - 10 - 10 - 10 - 10 - 10 - 10 - 10 - 1
[11]	ATTOCOURTLY THOU LAG NI
[16]	
1613	
120	Deficient and the Community of Mandaud Community Communi
	10000000000000000000000000000000000000
1 3	The first of the control of the cont
1	2

```
♥ CORRE KY$K$I$J$N$KRO$KRHO$T$JK
     ATABLE OF LAG 1-5 AND TOTAL AUTOCORRELATIONS FOR M/EK/1: FOR K IN KV
[1]
     ARHO MUST BE SET TO THE VECTOR OF INTENSITIES.
[23
[3]
      C+((N+FRHO),5)FI+1
[4]
      1+2
[5]
      M \in K \times (1-RHO) \div ((RHO \star 2) \times 1-K) + K \in K \vee [I]
[6]
      CT \leftarrow (RHO + 2) \times (K-1) \div (1-RHO) \times 2 \times K
[7]
      C[$1]+"1+KRHO+RHOxKRO+K+K+RHO
[8]
      C[;J]+C[;J-1]+((J-1);JK)x(K÷JK)x((RHO÷K)*J)xKRO*JK+J+KxJ-1
[9]
      48x1(6)J+J+1)
      C[ ; 1]+C[ ; 1]+KRO*K
[10]
      C[;2]+C[;2]+(KRO*2xK)X1+KRHO
[11]
[12]
      C[;3]+C[;3]+(KRO*3xK)x1+(2xKRHO)+(1+3xK)x(KRHO*2)+2xK
[13]
      C[;4]+C[;4]+(KRO*4xK)x1+(3xKRHO)+((T+1+4xK)x(KRHO*2)+K)
[14]
      [15]
      C[5] + C[5] + (KROx5xK)x1 + (4xKRHO) + (3x(T+1+5xK)x(KRHOx2) + 2xK)
      C[;5]+C[;5]+(KRO*5xK)x(Tx(T+1)x(KRHO*3)+3xK*2)
[16]
      C[;5]+C[;5]+(KRO*5xK)xTx(T+1)x(T+2)x(KRHO*4)+24xK*3
[17]
[18]
      10 2 f '
                                                              CV = ' ,3+K* 0.5
[19]
      □+
                                   K = ' , (0+K), '
                                  AUTOCORRELATION LAG N'
[20]
[21]
                                                                              TOTAL!
                                                                    5
[22]
[23]
      DEFORMATTENO, (Q(6,N)PM)xC,CT
[247
      +4x((FKV))1+I+1)
      VHPOOTS[[]]V
      V S-R HROOTS Q
      S-R+0.5\times1-(1-4\times R\times(1-R)\times(1-2\times Q)*2)*0.5
```

▼CONVERTIDI▼

▼ Q=CONVERT CV

[1] Q=(1-((-1+CV*2)+1+CV*2)*0.5)+2

▼

FORMAT 4 2 14 6 10 6 10 6 10 6 10 6

$\frac{4. \quad \text{TABLES}}{I(a). \, E_{k}/M/1}$

		K = 2	CV	= .707		
		AUTOC	ORRELATION	LAG N		
F:HO	N= 1	2	3	4	5	TOTAL
.05	004063	000095	000003	000000	000000	004161
.10	013101	000970	000093	000010	000001	014176
.15	023690	003202	000552	000107	000022	027580
.20	033818	006704	001674	000467	000139	042867
.25	042428	011000	003550	001274	000487	059073
.30	049059	015523	006048	002607	001193	075568
.35	053597	019778	008898	004406	002310	091944
.40	056124	023404	011787	006503	003789	107937
. 45	056818	026168	014429	008676	005495	123386
.50	055902	027951	016596	010700	007247	138197
.55	053602	028714	018127	012376	008855	152320
.60	050138	028478	018926	013553	010145	165740
.65	045711	027302	018952	014125	010980	178459
.70	.040497	025264	018203	014034	011260	190494
.75	034653	022455	016707	013258	010927	201871
.80	028312	018969	014511	011805	009954	212620
.85	021588	014897	011677	009705	008345	222774
.90	014576	010326	008268	007001	006123	232367
.95	007358	005336	004354	003747	003325	241434

		к 3	CV	= .577		
		AUTOC	OFFELATION	LAG N		
EHO	N== 1	2	3	4	5	TOTAL
.05	007023	000056	000001	000000	000000	007080
.10	024163	000982	000053	000003	000000	025202
.15	.044771	004107	000491	000066	000010	049446
.20	064047	009710	001890	000414	000097	076191
.25	-079537	016994	004596	001391	000450	103204
.30	090415	024783	008491	003236	001314	129239
.35	096773	032033	013098	005924	002846	153676
.40	099122	038021	017820	009191	005019	176260
. 45	.098119	042348	022109	012638	007626	196950
.50	.094417	044867	025550	015856	010357	215814
.55	088606	-,045602	027880	018494	012878	232976
.60	081191	044681	028974	020300	014891	248581
.65	072586	042285	028808	021121	016173	262778
.70	063127	038615	027430	020891	016578	275711
.75	053081	033872	024934	019610	016034	7.287512
.80	.042660	028244	021438	017329	014531	298302
.85	032028	021902	017072	014127	012103	308189
.90	021312	014997	011963	010101	008814	317269
.95	010612	007659	006235	005358	004749	325627

		к == 4	CV	= .500		
		AUTOC	ORRELATION	LAG N		
RHO	N= 1	2	3	4	5	TOTAL
			000000	000000		
.05	.009630	000032	000000	000000	000000	009662
.10	.034010	000874	000030	000001	000000	034916
.15	.063487	004394	.000401	000042	000005	068329
.20	.090330	011357	001852	000341	000068	103966
.25	110786	020772	004977	001339	000386	138430
.30	123978	030928	009722	003411	001277	170161
.35	130489	040291	015478	006596	002990	198648
.40	131448	047853	021420	010577	005538	223905
. 45	128055	053114	026794	014833	008479	246180
.50	121391	055950	031050	018816	012015	265803
.55	112346	056472	033862	022067	015110	283110
.60	101632	054917	035094	024266	017581	-,298415
.65	089794	051575	034752	025232	019146	311995
.70	077250	046742	032931	024902	019631	324090
.75	064314	040698	029779	023301	018963	334904
.80	051216	033696	025466	020510	017143	344612
.85	038129	025953	020169	016649	014233	353360
.90	025176	017656	014057	011852	010328	361273
.95	012446	008962	007288	006257	005543	368456

		K = 5	CV	* * * * * * * * * * * * * * * * * * * *		
RHO	N= 1	2	OFFELATION 3	LAG N	5	TOTAL.
.05 .10 .15 .20 .25 .30 .35 .40 .45 .50 .55 .60 .65 .70	012103 043103 080373 113348 137261 151475 157245 156355 150514 141146 129362 116002 116002 101684 086859 071849 056884	000019 000769 004482 012397 023407 035336 046246 054907 060764 063735 064016 061938 057873 052192 045229 045229	000000 000019 000332 001771 005131 010441 016998 023800 029937 034755 037886 039195 038713 036573 032965 028096	000000 000001 000028 000287 001266 003450 006943 011400 016208 020720 024394 024394 026862 027925 027526 025707 022576	000000 000003 000050 000335 001218 003020 005794 009282 013025 016513 019300 021062 021605 020856 018830	012123 043891 085219 127864 167529 202652 233064 259167 281539 300758 317341 331726 344275 355286 365002 373622
.85 .90 .95	042124 027680 013623	028592 019375 009798	022178 015407 007962	018279 012978 006833	015605 011300 006050	-373622 -381309 -388196 -394396

		K = 6	CV	= .408		
		AUTOC	ORRELATION	LAG N		
RHO	₩= 1	2	3	4	5	TOTAL
.05	014510	000012	000000	- 000000	- 000000	- 014500
				000000	000000	014522
.10	051660	000682	000012	000000	.000000	052355
.15	095788	.004499	000282	000020	000002	100591
.20	133699	013124	001691	000247	000039	148807
.25	159935	025371	005194	001199	000297	192101
.30	174339	038673	010905	003443	001162	-,229173
.35	178909	050752	018048	007142	003011	260321
. 40	176057	060214	025488	011935	005934	286370
. 45	167920	066473	032187	017144	009664	308200
.50	156187	069495	037415	022041	013698	326600
.55	142119	069549	040773	026024	017471	342217
.60	126630	067043	042129	028689	020489	355573
.65	110374	062418	041537	029823	022395	367085
.70	093807	056095	039160	029373	022984	377078
.75	077247	048452	035217	027398	022180	385815
.80	060911	039813	029948	024024	020008	393502
.85	044942	030447	023586	019419	016563	400308
.90	029434	020576	016349	013762	011976	406367
.95	014443	010380	008431	007233	006403	411790

		K = 7	CV	, m, m		
			OFFELATION	LAG N		
EHO	₩= 1	2	3	4	5	TOTAL
.05	016876	000008	000000	000000	000000	016884
.10	059795	000614	000009	000000	000000	060418
.15	109974	004490	000246	000015	000001	114726
.20	151842	013669	001620	000218	000032	167386
.25	179563	026905	005219	001142	-,000268	213186
.30	193623	041297	011226	003421	001115	251272
.35	196778	054288	018817	007265	002990	282474
.40	192003	064353	026747	012307	006016	308013
. 45	181784	070896	033876	017818	009923	329037
.50	168004	073925	039416	023009	014175	346490
.55	-,152024	073777	042943	027229	018164	361118
.60	134800	070920		030043	021358	373494
.65	-+116995	065849	043651	031231	023376	384061
.70	099062	059027		030744	024000	393163
.75		050862	036894	028651	023157	401065
.80	063918	041699	031322	025096	020878	407977
.85	047035	031823	024628	020261	017269	414064
.90	030731	021464	017044	014341	012475	419457
.95	015046	010808	008776	007528	006662	424264
• ,	.010040			.00/020	.00002	.424204

		к = 8	cv	= .354		
		AUTOC	ORRELATION	LAG N		
RHO	N== 1	2	3	4	5	TOTAL
.05	.019209	000006	000000	000000	.000000	.019215
.10	.067576	000560	.000006	000000	000000	068142
.15	123107	.004472	000219	000012	000001	127811
.20	168132	014098	001561	000197	000027	184019
.25	.196719	028144	005224	001095	000247	231507
.30	210098	043422	011460	003393	001075	269993
.35	211758	057139	019404	007345	002965	300847
.40	205162	067672	027721	012579	006066	325654
. 45	193076	074421	035191	018327	010108	345776
.50	177525	077438	040975	023748	014530	362278
.55	159932	077111	044633	028154	018688	375968
.60	141272	073963	046043	031086	022021	387454
.65	122207	068531	045292	032317	024126	397190
.70	103174	061310	042586	031801	024780	405524
.75	084461	052732	038190	029617	023907	412722
.80	066250	043158	032383	025921	021546	418988
.85	048652	032884	025431	020909	017811	424484
.90	031730	022147	017579	014786	012857	429338
.95	015510	011137	009041	007754	006861	433651

		к = 9	cv	= .333		
			ORRELATION	• 65 65 65		
EHO	≀≀ ≔ 1	2	3	4	5	TOTAL
.05	021517	000004	000000	000000	000000	021521
.10	075045	000517	000005	000000	000000	075567
. 15	135321	004453	000198	000010	000001	139983
.20	182850	014449	001511	000180	000023	199017
. 25	211841	029171	005222	001056	000230	247589
.30	-,224331	045180	011638	003365	001042	286066
.35	-,224489	059489	019867	007399	002939	316340
.40	-,216200	070394	028497	012785	006097	340313
. 45	-,202447	077298	036242	018723	010246	359521
.50	185357	080290	042223	024331	014803	375116
.55	166390	079808	045986	028886	019098	387947
.60	146525	-,076415	.047413	031913	022542	398636
. 65	126413	070684	046602	033179	024719	407646
.70	106478	063137	043778	032640	025397	415320
.75	086988	054224	039222	030384	024501	421920
.80	068109	044319	033225	026576	022075	427645
.85	049937	033726	026067	021422	.018240	432649
.90	032521	022688	018002	015137	013160	437057
.95	015876	011396	-,009250	007932	007018	440962

		K == 10	C	v = .316		
		AUTOC	ORRELATION	LAG N		
RHO	₩== 1	2	3	4	5	TOTAL
A						
.05	023799	000003	000000	000000	000000	023802
.10	082233	000482	000004	000000	000000	082718
.15	146722	004436	000181	000009	000000	151348
.20	196218	014745	001469	000167	000020	212622
.25	225272	030039	005215	001024	000216	261827
.30	236747	046662	011778	003339	001014	300022
.35	235441	061460	020241	007437	002916	329584
.40	225588	072667	029130	012947	006118	352691
. 45	210347	079690	037102	019039	010352	371011
.50	191911	082653	043245	024801	015020	385761
.55	171761	082033	047094	029480	019426	397813
.60	150872	078432	048533	032586	022963	407797
.65	129880	072450	047673	033881	025199	416171
.70	109191	064632	044751	033323	025898	423275
. 75	089057	055443	040063	031007	024984	429364
.80	069627	045265	033911	027108	022505	434630
.85	050984	034411	026585	021838	018588	439222
.90	033164	023127	018345	015422	013405	443256
.95	016173	011607	009420	008076	007146	446824

		K == 15		V = .258		
		AUTOC	ORRELATION	LAG N		
EHO	₩= 1	2	3	4	5	TOTAL
.05	-,034881	000001	000000	000000	000000	034882
.10	-,114628	000377	000002	000000	000000	115007
. 15	194135	004385	000135	000005	000000	198660
.20	-,248183	015754	001332	000129	000013	265413
.25	-,274828	032961	005169	000918	000176	314096
.30	280796	051586	012183	003236	000922	349121
.35	273152	067932	021384	007521	002824	374693
.40	257187	080052	031095	013408	006151	393801
. 45	236462	087392	039787	019986	010644	408430
.50	213272	090202	046440	026231	015655	419886
.55	189066	089098	050555	031303	020413	429040
.60	164745	084799	052029	034660	024243	436487
.65	.140856	.077999	051008	036048	026670	442640
.70	117724	069308	047777	035434	027436	447794
.75	095528	059238	042671	032934	026468	452165
.80	074351	048203	036033	028751	023827	455911
.85	054228	036532	028183	023123	019660	459152
.90	035150	024483	019405	016301	014160	461979
.95	017088	012255	009942	008521	007537	464465
. , .,	.017000	• 0 2	* VV77742	+000021	.00/55/	+404400

		K = 20	C	V = .224		
		AUTOC	ORRELATION	LAG N		
RHO	N= 1	2	3	4	5	TOTAL
.05	045453	000000	000000	000000	000000	.045454
.10	142251	000328	000001	.000000	.000000	142580
.15	230044	004377	.000114	.000003	000000	234538
.20	283961	016369	001261	000111	000011	301713
.25	306633	034659	005135	000863	000156	347482
.30	307663	054376	012379	003172	000872	378820
.35	295310	071533	021969	007543	002767	400892
.40	275240	084104	032117	013623	006150	416956
. 45	251064	091571	041192	020456	010773	429011
.50	225015	094260	.048115	026957	015962	438306
.55	198452	092865	052368	032238	020906	445645
.60	172186	088171	053856	035729	024892	451557
.65	146690	080921	052748	037168	027421	456403
.70	122224	071758	049351	036526	-,028226	460437
.75	098916	061217	044025	033931	027232	463837
.80	076813	049729	037133	029599	024508	466739
.85	055910	037629	029009	023786	020213	469238
.90	036175	025183	019951	016754	014549	471412
.95	017558	012589	010210	008750	007739	473318

BEST_AVAILABLE COTY

		K = 25	C	V = .200		
		AUTOC	OFFELATION	LAG N		
EHO	Max 1	2	3	4	5	TOTAL.
.05	055555	000000	000000	000000	000000	055555
.10	166142	000301	000001	000000	000000	166444
.15	-,258248	004388	000102	000003	000000	262741
.20	310127	016797	001217	000101	000009	328252
.25	328784	035781	005111	000828	000145	370681
.30	-,325757	056179	012494	003130	000841	398736
.35	309885	073831	022324	007550	002728	418023
.40	286915	086665	.032743	013746	006143	431816
. 45	260388	094194	042055	020735	010844	442035
.50	232440	096793	.049144	027395	016142	449838
.55	204340	095206	053483	032806	021201	455951
.60	176826	090259	054979	036381	025284	460846
.65	150309	082724	053817	037851	027878	464839
.70	125004	073266	.050317	037192	029706	468149
.75	101003	062433	044855	034539	027697	470931
.80	078323	050663	037805	030117	024924	473296
.85	056939	038301	029514	024191	020549	475330
.90	036801	025610	020284	017030	014786	477095
.95	017845	012792	010374	008890	007861	476639
						* * * * * * * * * * * * * * * * * * * *

		K = 50	c	V == .141		
		AUTOC	ORRELATION	LAG N		
RHO	₩= 1	2	3	4	5	TOTAL
.05	100000	000000	000000	000000	000000	100000
.10	249620	000256	000000	000000	000000	249876
.15	340405	004486	000081	000002	000000	344974
.20	377975	017855	001132	000082	000006	397052
.25	382215	038332	005060	-,000759	000123	426513
.30	367430	060140	012725	003035	000777	444396
.35	342429	078778	023048	007549	002643	456015
.40	312422	092112	034026	013978	006115	464007
. 45	280439	099722	043829	021288	010971	469764
.50	248219	102096	051263	028276	016492	474068
.55	216739	100081	055775	033957	021788	477387
.60	186524	094588	057288	037707	026074	480010
.65	157828	086448	056009	039245	028802	482129
.70	130752	076370	052296	038553	029683	483870
.75	105298	064928	046552	035781	028644	485324
.80	081421	052578	039180	031174	025769	486553
.85	059044	039672	030543	025015	021235	487604
.90	038078	026480	020963	017592	015268	488512
.95	018428	013206	010707	009173	008111	489304

AUTOCORRELATION LAG N	
RHO N= 1 2 3 4 5	TOTAL
AE	4////
	.166667
	.333261
	.408415
.20423115018539001093000074000005 -	.442827
.25414855039807005035000724000112 -	.460554
.30	.470757
.35360667081451023417007540002596 -	.477170
.40326388095013034684014086006093 -	.481486
.45291235102638044741021560011026 -	.484547
.50256610104873052353028718016661 -	.486810
	.488540
.60191593096832058473038382026470 -	.489899
.65161735088371057135039956029269 -	.490990
.70133723077967053310039247030178 -	.491883
.75107510066209047421036414029126 -	.492626
.80083011053558039882031713026200 -	.493252
	.493787
	.494247
	.494648

		K == 10	000	cv = .0	10	
		DOTUA	ORRELATION	LAG N		
EHO	N== 1	2	3	4	5	TOTAL
.05	.490196	000000	000000	000000	000000	490196
.10	497262	000249	000000	000000	000000	497511
.15	493989	004828	000065	000001	000000	498883
.20	478856	019375	001056	000066	000004	499358
.25	452310	041442	005012	000689	000102	499574
.30	418181	064665	012958	002930	-,000711	499692
.35	380226	084246	023787	007526	002546	499763
.40	341109	098013	035347	014187	006066	499809
. 45	302480	105632	045661	021826	011075	499842
.50	265273	107710	053452	029157	016823	499866
.55	229967	105203	058143	035122	022366	499884
.60	196765	099108	059669	039057	026864	499898
.65	165704	090317	058268	040668	029735	499909
.70	136731	079580	054331	039943	030674	499918
.75	109743	067499	048295	037050	029609	499926
.80	084612	054544	040588	032253	026631	499932
.85	061203	041077	031596	025857	021933	499938
.90	039383	027370	021656	018166	015759	499942
.95	019022	013628	011047	009463	008365	499946

			00000	CV ==	.001	
		AUTOC	OFFELATION	LAG N		
EHO	₩== 1	2	3	4	5	TOTAL
.05		.000000	.000000	000000	.000000	499900
.10	499725	000250	.000000		000000	-,499975
.15	495092	004831	-,000065	000001	000000	499989
.20	479483	019385	001056	000066	000004	499994
.25	452715	041459	005012	000689	000102	499996
.30	418462	064689	012959	002930	000711	499997
.35	380430	084275	023791	007526	002546	499998
.40	341261	098044	035353	014187	006066	-,499998
. 45	302595	105662	045670	021829	011075	-,499998
.50	265361	107738	053463	029161	016825	499999
.55	230036	105230	058155	035128	022369	499999
.60	196818	099131	059681	039064	026868	499999
.65	165745	090337	058279	040676	029740	499999
.70	136762	079597	054341	039950	030679	499999
.75	109765	067512	048304	037056	029613	499999
.80	084628	054554	040595	032259	026635	499999
.85	061214	041084	031602	025861	021937	499999
.90	039390	027374	021659	018169	015762	499999
.95	019025	013630	011048	009464	008366	499999

I(b). H/M/1

		Q = .345	59 (ev = 1.10		
		AUTOCO	DRRELATION	LAG N		
RHO	N= 1	2	3	4	5	TOTAL
.05	.000178	.000018	.000002	.000000	.000000	.000198
.10	.000645	.000120	.000027	.000007	.000002	.000801
.15	.001315	.000341	.000107	.000037	.000014	.001822
.20	.002119	.000683	.000264	.000112	.000050	.003275
.25	.002998	.001131	.000508	.000250	.000129	.005176
.30	.003901	.001660	.000835	.000458	.000264	.007543
.35	.004786	.002240	.001232	.000734	.000460	.010396
.40	.005614	.002840	.001677	.001070	.000715	.013755
. 45	.006351	.003427	.002146	.001447	.001020	.017643
.50	.006964	.003969	.002610	.001842	.001357	.022087
.55	.007423	.004434	.003039	.002229	.001703	.027114
.60	.007701	.004790	.003401	.002577	.002031	.032756
.65	.007770	.005007	.003666	.002857	.002311	.039045
.70	.007602	.005053	.003801	.003034	.002510	.046021
.75	.007169	.004899	.003772	.003076	.002595	.053724
.80	.006444	.004514	.003548	.002947	.002528	.062201
.85	.005397	.003865	.003094	.002612	.002275	.071505
.90	.003996	.002920	.002376	.002035	.001795	.081695
.95	.002209	.001644	.001358	.001178	.001051	.092835

		Q = .28	77	CV = 1.20		
		AUTOC	DERELATION			
EHO	ผ≕ 1	2	3	4	5	TOTAL
AF:	000001	000070	00000	000001	00000	000710
.05	.000284	.000030	.000004	.000001	.000000	.000319
.10	.001030	.000203	.000049	.000013	.000004	.001300
.15	.002108	.000575	.000189	.000069	.000026	.002985
.20	.003410	.001150	.000464	.000205	.000096	.005420
. 25	.004847	.001904	.000888	.000451	.000242	.008656
.30	.006343	.002798	.001454	.000821	.000487	.012750
.35	.007832	.003787	.002141	.001310	.000840	.017769
.40	.009254	.004820	.002917	.001902	.001298	.023783
. 45	.010552	.005846	.003739	.002569	.001843	.030876
.50	.011673	.006812	.004564	.003273	.002447	.039138
.55	.012564	.007665	.005340	.003971	.003072	.048676
.60	.013173	.008351	.006016	.004613	.003673	.059608
.65	.013444	.008813	.006535	.005144	.004197	.072072
.70	.013318	.008992	.006837	.005506	.004588	.086227
.75	.012731	.008825	.006859	.005634	.004781	.102261
.80	.011613	.008240	.006531	.005457	.004706	.120391
.85	.009883	.007161	.005775	.004900	.004284	.140881
.90	.007448	.005499	.004504	.003873	.003428	.164045
.95	.004197	.003153	.002618	.002279	.002039	.190262

		Q = .246	8 (= 1.30				
	AUTOCORRELATION LAG N							
RHO	N= 1	2	3	4	5	TOTAL		
.05	.000344	.000039	.000005	.000001	.000000	.000390		
.10	.001251	.000259	.000065	.000018	.000005	.001602		
• 15	.002566	.000731	.000251	.000095	.000038	.003709		
.20	.004163	.001460	.000611	.000280	.000135	.006793		
. 25	.005939	.002417	.001164	.000610	.000337	.010951		
.30	.007808	.003556	.001901	.001101	.000670	.016289		
.35	.009691	.004823	.002796	.001750	.001147	.022931		
.40	.011517	.006157	.003809	.002534	.001761	.031018		
. 45	.013220	.007499	.004891	.003418	.002492	.040710		
.50	.014733	.008784	.005987	.004359	.003303	.052194		
.55	.015988	.009946	.007035	.005300	.004147	.065687		
.60	.016916	.010916	.007970	.006180	.004969	.081442		
.65	.017437	.011618	.008717	.006929	.005701	.099762		
.70	.017467	.011969	.009197	.007468	.006267	.121013		
.75	.016906	.011876	.009317	.007707	.006580	.145640		
.80	.015635	.011230	.008974	.007545	.006538	.174195		
.85	.013513	.009901	.008042	.006859	.006021	.207375		
.90	.010362	.007730	.006370	.005503	.004887	.246068		
.95	.005956	.004516	.003771	.003294	.002955	.291432		

		Q = .215	5.3	= 1.40				
AUTOCORRELATION LAG N								
F:HO	พ= 1	2	3	4	5	TOTAL.		
.05	.000375	.000045	.000007	.000001	.000000	.000428		
.10	.001366	.000295	.000077	.000022	.000007	.001771		
.15	.002806	.000830	.000296	.000116	.000048	.004134		
.20	.004564	.001657	.000716	.000338	.000169	.007634		
.25	.006532	.002742	.001358	.000731	.000414	.012412		
.30	.008619	.004037	.002213	.001313	.000817	.018629		
.35	.010743	.005484	.003251	.002077	.001388	.026472		
.40	.012830	.007020	.004430	.003000	.002121	.036157		
. 45	.014808	.008580	.005696	.004044	.002991	.047936		
.50	.016605	.010094	.006990	.005159	.003958	.062105		
.55	.018146	.011491	.008243	.006285	.004971	.079014		
.60	.019348	.012691	.009383	.007353	.005967	.099086		
.65	.020120	.013607	.010325	.008282	.006869	.122836		
.70	.020355	.014141	.010975	.008983	.007589	.150905		
.75	.019922	.014173	.011219	.009345	.008024	.184108		
.80	.018662	.013561	.010923	.009239	.008044	.223502		
.85	.016368	.012122	.009916	.008501	.007493	.270483		
.90	.012768	.009619	.007977	.006921	.006167	.326948		
.95	.007488	.005729	.004810	.004218	.003795	.395529		

		Q = .189	9 0	V = 1.50		
		AUTOCO	RRELATION	LAG N		
RHO	N= 1	2	3	4	5	TOTAL.
.05	.000388	.000048	.000007	.000001	.000000	.000445
.10	.001413	.000316	.000086	.000026	.000008	.001854
• 15	.002907	.000889	.000326	.000132	.000056	.004356
.20	.004737	.001771	.000787	.000382	.000195	.008106
.25	.006797	.002931	.001488	.000820	.000475	.013284
.30	.008995	.004319	.002420	.001466	.000930	.020105
.35	.011251	.005875	.003553	.002311	.001572	.028819
.40	.013491	.007536	.004841	.003332	.002392	.039721
. 45	.015643	.009238	.006232	.004488	.003364	.053161
.50	.017633	.010908	.007663	.005728	.004446	.069550
.55	.019384	.012474	.009066	.006990	.005584	.089385
.60	.020807	.013852	.010362	.008200	.006712	.113277
.65	.021803	.014950	.011463	.009275	.007750	.141983
.70	.022250	.015656	.012265	.010116	.008602	.176478
.75	.021998	.015836	.012642	.010601	.009152	.218039
.80	.020850	.015317	.012432	.010577	.009253	.268388
.85	.018543	.013873	.011426	.009846	.008714	.329912
.90	.014707	.011184	.009333	.008134	.007272	.406023
.95	.008802	.006794	.005736	.005050	.004555	.501780

		Q = .169	70 (CV = 1.60		
		AUTOCO	DRRELATION	LAG N		
EHO	N= 1	2	3	4	5	TOTAL
.05	.000388	.000050	.000008	.000001	.000000	.000448
.10	.001416	.000327	.000092	.000028	.000009	.001878
.15	.002917	.000917	.000346	.000143	.000063	.004441
.20	.004762	.001827	.000832	.000413	.000216	.008322
.25	.006846	.003023	.001569	.000883	.000522	.013738
.30	.009083	.004456	.002547	.001571	.001015	.020954
.35	.011394	.006070	.003736	.002471	.001707	.030285
.40	.013709	.007801	.005092	.003556	.002588	.042102
. 45	.015958	.009586	.006561	.004787	.003631	.056851
.50	.018069	.011355	.008082	.006112	.004794	.075065
.55	.019963	.013036	.009587	.007468	.006022	.097394
.60	.021553	.014545	.010998	.008783	.007248	.124640
.65	.022735	.015787	.012224	.009972	.008391	.157828
.70	.023381	.016646	.013157	.010930	.009352	.198298
.75	.023327	.016977	.013662	.011530	.010008	.247864
.80	.022349	.016587	.013561	.011603	.010198	.309056
.85	.020137	.015209	.012609	.010920	.009703	.385531
.90	.016231	.012453	.010454	.009151	.008209	.482784
.95	.009914	.007715	.006549	.005788	.005237	.609464

		Q = .15	15	cv = 1.70		
		AUTOCO	DERELATION	LAG N		
RHO	₩= 1	2	3	4	5	TOTAL
.05	.000381	.000050	.000008	.000001	.000000	.000442
.10	.001392	.000330	.000095	.000030	.000010	.001863
.15	.002871	.000925	.000357	.000151	.000068	.004433
.20	.004692	.001841	.000856	.000434	.000232	.008360
.25	.006757	.003046	.001611	.000923	.000556	.013896
.30	.008983	.004493	.002613	.001638	.001075	.021351
.35	.011297	.006125	.003830	.002570	.001800	
.40	.013632	.007885	.005221	.003695	.001800	.031100
.45	.015920	.009710	.006733	.004971	.003812	
.50	.018094	.011533	.008307	.006349	.005028	.059364
.55	.020077	.013285	The second secon			.079073
			.009877	.007768	.006317	.103518
.60	.021785	.014884	.011367	.009156	.007613	.133702
.65	.023114	.016236	.012686	.010428	.008834	.170916
.70	.023934	.017224	.013727	.011482	.009882	.216893
.75	.024073	.017697	.014350	.012185	.010631	.274030
.80	.023293	.017451	.014366	.012359	.010912	.345783
.85	.021245	.016188	.013506	.011754	.010485	.437338
.90	.017390	.013453	.011359	.009986	.008989	.556851
.95	.010838	.008500	.007254	.006435	.005840	.717921

		a = .136	56 (= 1.80		
		AUTOCO	DERELATION	LAG N		
EHO	N ≡ 1	2	3	4	5	TOTAL
.05	.000370	.000050	.000008	.000002	.000000	.000430
.10	.001351	.000328	.000097	.000031	.000011	.001824
.15	.002788	.000918	.000362	.000156	.000071	.004364
.20	.004563	.001826	.000865	.000446	.000242	.008276
. 25	.006581	.003021	.001625	.000947	.000579	.013845
.30	.008764	.004458	.002633	.001675	.001115	.021421
.35	.011044	.006083	.003858	.002623	.001860	.031434
.40	.013359	.007841	.005260	.003766	.002806	.044406
. 45	.015646	.009673	.006788	.005066	.003923	.060972
.50	.017840	.011518	.008386	.006472	.005171	.081901
.55	.019870	.013306	.009991	.007926	.006497	.108143
.60	.021654	.014961	.011530	.009361	.007838	.140894
. 65	.023092	.016392	.012916	.010693	.009115	.181716
.70	.024056	.017483	.014042	.011821	.010230	.232736
.75	.024373	.018085	.014769	.012613	.011059	.296970
.80	.023796	.017984	.014902	.012888	.011427	.378885
.85	.021951	.016864	.014155	.012377	.011082	.485420
.90	.018235	.014217	.012070	.010656	.009624	.627959
.95	.011594	.009160	.007856	.006996	.006367	.826550

		Q = .123	38	CV = 1.90		
		AUTOCO	DERELATION	LAG N		
R:HO	N== 1	2	3	4	5	TOTAL
.05	.000356	.000049	.000008	.000002	.000000	.000416
.10	.001300	.000322	.000097	.000032	.000011	.001769
.15	.002685	.000901	.000362	.000159	.000074	.004253
.20	.004399	.001791	.000863	.000452	.000249	.008110
.25	.006351	.002963	.001618	.000956	.000593	.013649
.30	.008471	.004374	.002619	.001688	.001137	.021257
.35	.010695	.005973	.003836	.002639	.001893	.031415
.40	.012963	.007709	.005231	.003786	.002849	.044716
. 45	.015219	.009525	.006756	.005091	.003978	.061885
.50	.017401	.011364	.008356	.006506	.005241	.083807
.55	.019445	.013162	.009974	.007976	.006586	.111574
.60	.021271	.014846	.011538	.009435	.007952	.146570
.65	.022785	.016329	.012967	.010806	.009265	.190620
.70	.023864	.017501	.014157	.011990	.010430	.246243
.75	.024337	.018213	.014972	.012857	.011325	.317086
.80	.023956	.018252	.015217	.013225	.011775	.408689
.85	.022333	.017289	.014594	.012818	.011519	.529926
.90	.018813	.014774	.012609	.011178	.010128	.695942
.95	.012198	.009703	.008363	.007475	.006822	.934816

		Q = .112	27	CV = 2.00		
		AUTOC	DESELATION	LAG N		
EHO	₩== 1	2	3	4	5	TOTAL
.05	.000341	.000048	.000008	.000002	.000000	.000399
.10	.001244	.000314	.000096	.000032	.000012	.001705
.15	.002571	.000877	.000358	.000160	.000075	.004117
.20	.004215	.001743	.000852	.000453	.000253	.007890
. 25	.006093	.002883	.001596	.000955	.000599	.013354
.30	.008137	.004258	.002581	.001683	.001147	.020926
.35	.010288	.005818	.003779	.002628	.001904	.031136
.40	.012492	.007516	.005154	.003766	.002860	.044645
. 45	.014697	.009300	.006660	.005063	.003989	.062264
.50	.016845	.011116	.008247	.006472	.005252	.084990
.55	.018877	.012903	.009859	.007942	.006602	.114053
.60	.020718	.014594	.011430	.009410	.007979	.151017
.65	.022281	.016108	.012882	.010802	.009312	.197956
.70	.023447	.017338	.014119	.012024	.010512	.257771
.75	.024053	.018142	.015006	.012952	.011459	.334742
.80	.023854	.018309	.015353	.013406	.011983	.435516
.85	.022455	.017506	.014856	.013104	.011819	.571046
.90	.019168	.015154	.012998	.011567	.010515	.760712
.95	.012669	.010142	.008782	.007877	.007210	1.042245

		Q = .090)7	cv = 2.25		
		AUTOCO	RRELATION	LAG N		
RHO	พ= 1	2	3	4	5	TOTAL
.05	.000301	.000044	.000008	.000002	.000000	.000355
.10	.001099	.000287	.000091	.000032	.000012	.001529
.15	.002274	.000802	.000338	.000156	.000076	.003727
.20	.003733	.001593	.000802	.000439	.000252	.007221
.25	.005407	.002635	.001499	.000921	.000593	.012374
.30	.007239	.003893	.002420	.001616	.001127	.019668
.35	.009180	.005327	.003542	.002518	.001863	.029732
.40	.011187	.006896	.004833	.003604	.002790	.043379
. 45	.013215	.008556	.006253	.004843	.003883	.061628
.50	.015220	.010263	.007760	.006195	.005109	.085741
.55	.017151	.011967	.009306	.007617	.006424	.117268
.60	.018950	.013612	.010837	.009054	.007779	.158177
.65	.020543	.015129	.012288	.010443	.009112	.211103
.70	.021831	.016430	.013574	.011705	.010346	.279829
.75	.022672	.017390	.014580	.012730	.011376	.370148
.80	.022843	.017817	.015130	.013352	.012044	.491507
.85	.021965	.017388	.014933	.013301	.012096	.660433
.90	.019321	.015499	.013443	.012072	.011055	.908543
.95	.013364	.010849	.009493	.008586	.007912	1.304606

		Q = .074	15	cv = 2.50		
			DERELATION			
RHO	₩= 1	2	3	4	5	TOTAL.
.05	.000263	.000040	.000007	.000002	.000000	.000312
.10	.000963	.000259	.000084	.000030	.000011	.001355
.15	.001993	.000721	.000311	.000147	.000073	.003328
.20	.003276	.001431	.000737	.000412	.000242	.006508
.25	.004752	.002368	.001375	.000862	.000566	.011271
.30	.006373	.003500	.002218	.001509	.001072	.018140
.35	.008099	.004794	.003246	.002347	.001766	.027821
.40	.009894	.006214	.004430	.003357	.002638	.041257
. 45	.011723	.007727	.005737	.004510	.003667	.059662
.50	.013549	.009292	.007132	.005773	.004822	.084543
.55	.015332	.010872	.008574	.007108	.006067	.117744
.60	.017024	.012418	.010019	.008471	.007358	.161572
.65	.018566	.013876	.011413	.009808	.008645	.219124
.70	.019878	.015173	.012685	.011052	.009860	.294925
. 75	.020841	.016203	.013738	.012111	.010916	.396131
.80	.021265	.016802	.014421	.012842	.011676	•534871
.85	.020806	.016677	.014467	.012995	.011904	.733388
.90	.018777	.015244	.013348	.012082	.011139	1.037054
.95	.013543	.011121	.009819	.008947	.008295	1.554436

		Q = .062	23	CV = 2.75				
AUTOCORRELATION LAG N								
RHO	N= 1	2	3	4	5	TOTAL		
.05	.000230	.000036	.000007	.000001	.000000	.000274		
.10	.000843	.000231	.000077	.000028	.000011	.001197		
.15	.001745	.000644	.000283	.000136	.000069	.002959		
.20	.002871	.001277	.000669	.000380	.000227	.005829		
.25	.004169	.002113	.001247	.000794	.000529	.010187		
.30	.005598	.003124	.002011	.001388	.001000	.016576		
.35	.007126	.004283	.002942	.002156	.001643	.025760		
.40	.008721	.005558	.004016	.003081	.002451	.038791		
. 45	.010356	.006921	.005205	.004140	.003404	.057063		
.50	.012000	.008341	.006479	.005303	.004476	.082324		
.55	.013621	.009783	.007804	.006537	.005634	.116684		
.60	.015181	.011211	.009143	.007806	.006842	.162734		
.65	.016633	.012579	.010452	.009065	.008058	.223930		
.70	.017911	.013828	.011673	.010259	.009225	.305408		
.75	.018921	.014872	.012727	.011311	.010270	.415503		
.80	.019501	.015572	.013483	.012099	.011076	•568796		
.85	.019355	.015672	.013710	.012405	.011436	.793061		
.90	.017854	.014638	.012921	.011775	.010921	1.148005		
.95	.013374	.011087	.009865	.009046	.008434	1.788751		

		Q = .052	28	= 3.00		
		AUTOCO	DESELATION	LAG N		
EHO	₩::: 1	2	3	4	5	TOTAL
.05	.000202	.000032	.000006	.000001	.000000	.000242
.10	.000739	.000206	.000069	.000026	.000010	.001058
.15	.001532	.000574	.000256	.000125	.000064	.002629
.20	.002522	.001138	.000604	.000348	.000211	.005212
. 25	.003665	.001883	.001126	.000726	.000490	.009181
.30	.004926	.002785	.001814	.001267	.000923	.015084
.35	.006277	.003819	+002653	.001966	.001514	.023724
.40	.007693	.004961	.003622	.002808	.002256	.036245
. 45	.009150	.006184	.004698	.003773	.003131	.054211
.50	.010624	.007464	.005854	.004835	.004116	.079603
.55	.012089	.008772	.007062	.005966	.005183	.114784
.60	.013512	.010078	.008292	.007136	.006302	.162575
.65	.014857	.011344	.009505	.008307	.007436	.226707
.70	.016073	.012523	.010657	.009434	.008540	.312797
.75	.017080	.013546	.011681	.010453	.009550	.430190
.80	.017748	.014295	.012469	.011262	.010370	.595650
.85	.017824	.014554	.012822	.011673	.010821	.842112
.90	.016751	.013845	.012304	.011279	.010515	1.243513
.95	.012982	.010847	.009714	.008957	.008392	2.006050

		Q = .045	53	cv = 3.25		
		AUTOCO	DERELATION	LAG N		
RHO	N= 1	2	3	4	5	TOTAL
.05	.000178	.000028	.000006	.000001	.000000	.000213
.10	.000651	.000184	.000063	.000024	.000009	.000938
.15	.001350	.000512	.000231	.000114	.000059	.002341
.20	.002224	.001015	.000545	.000317	.000194	.004665
.25	.003233	.001679	.001014	.000660	.000450	.008272
.30	.004349	.002484	.001634	.001152	.000846	.013708
.35	.005547	.003408	.002390	.001786	.001387	.021796
.40	.008806	.004430	.003263	.002551	.002065	.033758
. 45	.008106	.005528	.004234	.003427	.002865	.051314
.50	.009427	.006680	.005281	.004394	.003767	.076677
.55	.010746	.007863	.006379	.005427	.004746	.112451
.60	.012039	.009052	.007501	.006499	.005776	.161649
.65	.013275	.010215	.008619	.007581	.006826	.228193
.70	.014413	.011316	.009694	.008633	.007859	.318083
.75	.015391	.012296	.010671	.009605	.008822	.441510
.80	.016100	.013060	.011462	.010410	.009634	.617166
.85	.016328	.013425	.011898	.010889	.010142	.882696
.90	.015590	.012972	.011595	.010682	.010003	1.325703
. 95	.012454	.010475	.009432	.008740	.008223	2.205913

		Q = .039	27 (ev = 3.50		
		2	DESELATION	(1) V (1) (1)		
EHO	N= 1	2	3	4	5	TOTAL.
.05	.000157	.000025	.000005	.000001	.000000	.000189
.10	.000577	.000165	.000057	.000022	.000009	.000835
.15	.001196	.000458	.000209	.000104	.000055	.002090
.20	.001970	.000907	.000492	.000289	.000178	.004185
.25	.002866	.001501	.000915	.000600	.000412	.007464
.30	.003857	.002222	.001473	.001046	.000774	.012462
.35	.004923	.003049	.002154	.001621	.001268	.020013
.40	.006045	.003965	.002942	.002315	.001887	.031398
. 45	.007207	.004952	.003819	.003111	.002617	.048482
.50	.008392	.005990	+004766	.003990	.003440	.073712
.55	.009581	.007059	.005762	.004931	.004336	.109927
.60	.010753	.008139	.006785	.005912	.005281	.160291
.65	.011885	.009204	.007811	.006907	.006250	.228858
.70	.012942	.010224	.008808	.007884	.007211	.321915
.75	.013875	.011152	.009731	.008801	.008120	.450374
.80	.014594	.011908	.010506	.009586	.008910	.634611
.85	.014922	.012340	.010991	.010104	.009449	.916513
.90	.014442	.012085	.010853	.010042	.009441	1.396529
.95	.011853	.010024	.009068	.008438	.007969	2.388657

		Q = .034	14	cv = 3.75				
AUTOCORRELATION LAG N								
RHO	N= 1	2	3	4	5	TOTAL.		
.05	.000140	.000023	.000005	.000001	.000000	.000169		
.10	.000513	.000148	.000051	.000020	.000008	.000746		
• 15	.001065	.000411	.000189	.000095	.000050	.001874		
.20	.001754	.000814	.000444	.000263	.000163	.003765		
.25	.002553	.001347	.000826	.000546	.000377	.006749		
.30	.003437	.001993	.001330	.000950	.000708	.011344		
.35	.004389	.002736	.001945	.001473	.001159	.018383		
.40	.005394	.003560	.002657	.002103	.001723	.029193		
. 45	.006435	.004448	.003450	.002825	.002389	.045773		
.50	.007501	.005385	.004308	.003625	.003141	.070802		
•55	.008573	.006353	.005213	.004482	.003960	.107354		
.60	.009637	.007334	.006145	.005379	.004827	.158712		
.65	.010671	.008309	.007085	.006293	.005719	.229000		
.70	.011649	.009251	.008006	.007198	.006610	.324728		
.75	.012529	.010122	.008872	.008058	.007463	.457418		
.80	.013239	.010856	.009620	.008813	.008221	.648916		
.85	.013630	.011326	.010131	.009349	.008774	.944901		
.90	.013346	.011220	.010118	.009397	.008865	1.457719		
.95	.011220	.009532	.008658	.008085	.007660	2.555068		

		$\alpha = .030$)3	cv = 4.00		
		AUTOCO	DERELATION	LAG N		
EHO	H== 1	2	3	4	5	TOTAL
.05	.000125	.000021	.000004	.000001	.000000	.000151
.10	.000459	.000133	.000047	.000018	.000007	.000670
.15	.000952	.000370	.000171	.000086	.000046	.001686
.20	.001570	.000733	.000402	.000239	.000150	.003399
. 25	.002285	.001213	.000748	.000497	.000345	.006119
.30	.003077	.001795	.001204	.000865	.000648	.010346
.35	.003932	.002464	.001761	.001340	.001059	.016905
.40	.004834	.003207	.002405	.001913	.001574	.027153
. 45	.005771	.004010	.003124	.002570	.002183	.043214
.50	.006732	.004857	.003903	.003298	.002870	.067999
.55	.007702	.005735	.004726	.004081	.003620	.104816
.60	.008668	.006628	.005577	.004901	.004414	.157039
.65	.009613	.007519	.006438	.005740	.005235	.228814
.70	.010516	.008388	.007288	.006576	.006060	.326818
.75	.011342	.009202	.008096	.007379	.006857	.463093
.80	.012030	.009906	.008811	.008099	.007580	.660771
.85	.012458	.010394	.009331	.008640	.008133	.968906
.90	.012320	.010399	.009411	.008768	.008296	1.510755
.95	.010582	.009025	.008226	.007705	.007321	2.706204

		Q = .024	41	CV = 4.50		
		AUTOC	DRRELATION	LAG N		
RHO	N= 1	2	3	4	5	TOTAL
.05	.000102	.000017	.000003	.000001	.000000	.000123
.10	.000372	.000109	.000039	.000015	.000006	.000546
.15	.000773	.000303	.000142	.000072	.000039	.001381
.20	.001274	.000601	.000333	.000200	.000126	.002799
.25	.001856	.000994	.000619	.000415	.000291	.005074
.30	.002501	.001471	.000995	.000721	.000544	.008665
.35	.003197	.002021	.001455	.001116	.000889	.014363
.40	.003934	.002632	.001989	.001593	.001321	.023556
. 45	.004702	.003292	.002585	.002141	.001831	.038574
.50	.005491	.003992	.003231	.002749	.002407	.062800
.55	.006292	.004720	.003916	.003404	.003038	.100013
.60	.007094	.005464	.004628	.004093	.003709	.153694
. 65	.007887	.006212	.005354	.004802	.004405	.227918
.70	.008654	.006951	.006077	.005516	.005111	.329575
.75	.009374	.007656	.006778	.006213	.005804	.471543
.80	.010003	.008292	.007418	.006857	.006450	.679069
.85	.010455	.008779	.007927	.007379	.006981	1.006871
.90	.010508	.008925	.008123	.007607	.007233	1.597184
.95	.009356	.008029	.007358	.006927	.006613	2.967441

		Q = .019	76	CV = 5.00		
		AUTOCO	DESELATION	LAG N		
EH0	<i>i</i> ·ℓ== 1	2	3	4	5	TOTAL
.05	.000084	.000014	.000003	.000001	.000000	.000102
.10	.000307	.000091	.000032	.000013	.000005	.000453
.15	.000638	.000252	.000119	.000061	.000033	.001148
.20	.001052	.000500	.000279	.000169	.000107	.002336
.25	.001533	.000827	.000518	.000349	.000246	.004258
.30	.002067	.001224	.000833	.000607	.000461	.007329
.35	.002643	.001681	.001218	.000939	.000752	.012295
.40	.003254	.002190	.001665	.001341	.001117	.020537
. 45	.003892	.002741	.002164	.001802	.001548	.034548
.50	.004549	.003326	.002706	.002314	.002036	.058174
.55	.005218	.003936	.003283	.002867	.002571	.095685
.60	.005892	.004562	.003884	.003450	.003141	.150579
. 65	.006562	.005196	.004499	.004054	.003735	.226744
.70	.007217	.005826	.005118	.004666	.004341	.331182
.75	.007841	.006437	.005724	.005269	.004943	.477410
.80	.008405	.007001	.006292	.005840	.005515	.692321
.85	.008846	.007464	.006769	.006326	.006008	1.035112
.90	.009000	.007681	.007020	.006601	.006299	1.663610
.95	.008246	.007110	.006543	.006183	.005924	3.181976

		Q = .016	53	cv = 5.50		
		AUTOCO	DERELATION	LAG N		
F:HO	₩== 1	2	3	4	5	TOTAL
.05	.000070	.000012	.000002	.000001	.000000	.000085
.10	.000258	.000077	.000027	.000011	.000005	.000381
.15	.000535	.000213	.000101	.000052	.000028	.000967
.20	.000882	.000421	.000236	.000143	.000092	.001974
.25	.001286	.000697	.000438	.000297	.000211	.003614
.30	.001734	.001032	.000705	.000516	.000394	.006261
.35	.002218	.001417	.001031	.000799	.000642	.010608
.40	.002731	.001847	.001409	.001140	.000954	.018006
. 45	.003268	.002312	.001833	.001532	.001322	.031066
.50	.003823	.002807	.002293	.001968	.001738	.054087
•55	.004389	.003324	.002783	.002439	.002195	.091850
.60	.004960	.003856	.003295	.002938	.002683	.147793
. 65	.005531	.004397	.003821	.003455	.003194	.225539
.70	.006094	.004939	.004354	.003982	.003717	.332145
.75	.006637	.005469	.004880	.004507	.004241	.481639
.80	.007138	.005968	.005382	.005011	.004747	.702207
.85	.007552	.006396	.005819	.005456	.005197	1.056617
.90	.007758	.006644	.006093	.005746	.005499	1.715471
.95	.007273	.006293	.005810	.005507	.005291	3.358578

		G = .013	ı ,	EV = 6.00		
		• V.I. v.) DERELATION			
EHO	N== 1	2	3	4	5	TOTAL
.05	.000060	.000010	.000002	.000000	.000000	.000073
.10	.000219	.000065	.000024	.000009	.000004	.000324
. 15	.000454	.000181	.000086	.000045	.000024	.000825
.20	.000750	.000359	.000202	.000123	.000079	.001688
. 25	.001092	.000594	.000375	.000255	.000182	.003100
.30	.001473	.000880	.000604	.000443	.000339	.005398
. 35	.001885	.001209	.000883	.000484	.000553	.009222
.40	.002322	.001576	.001206	.000979	.000822	.015878
. 45	.002780	.001973	.001569	.001316	.001138	.028050
.50	.003253	+002396	.001964	.001690	.001497	.050478
.55	.003737	.002839	.002384	.002096	.001891	.088473
.60	.004227	.003296	.002825	.002525	.002313	.145347
. 65	.004718	.003762	.003279	.002973	.002755	.224404
.70	.005205	.004231	.003740	.003430	.003210	.332736
. 75	.005679	.004693	.004200	.003889	.003668	.484782
.80	.006124	.005135	.004643	.004334	.004115	.709767
.85	.006506	.005525	.005040	.004737	.004523	1.073330
.90	.006733	.005783	.005317	.005026	.004821	1.756577
.95	.006432	.005580	.005165	.004907	.004725	3.504648

		Q = .010)1 DERELATION	CV = 7.00		
EH0	N= 1	2	3	4	5	TOTAL
.05	.000045	.000008	.000002	.000000	.000000	.000054
.10	.000163	.000049	.000018	.000007	.000003	.000242
.20	.000559	.000269	.000153	.000094	.000060	.001270 .002345
.30	.001099	.000660	.000455	.000336	.000258	.004115
.40	.001734	.001182	.000910	.000741	.000625	.012552
.45	.002077 .002432	.001482	.001183	.000996	.000866	.023144
.55	.002796	.002134	.001800	.001589	.001439	.082886
.65	.003540	.002835	.002481 .002835	.002258	.002100	.222462
.75	.004280	.003552	.003190	.002965	.002807	.489050 .720373
.85	.004950	.004221	.003865	.003645	.003493	1.097191
.90 .95	.005179 .005087	.004466	.004121	.003909	.003762	1.816511 3.728008

		a = .007	78	CV = 8.00		
		AUTOCO	MOTTALBARD	LAG N		
EHO	<i>1</i> == 1	2	3	4	5	TOTAL.
.05	.000034	.000006	.000001	.000000	.000000	.000042
.10	.000126	.000038	.000014	.000006	.000002	.000188
.15	.000262	.000106	.000051	.000026	.000015	.000480
.20	.000432	.000209	.000119	.000073	.000047	.000988
.25	.000630	.000346	.000220	.000151	.000108	.001830
.30	.000850	.000512	.000354	.000262	.000202	.003229
.35	.001089	.000704	.000518	.000406	.000330	.005642
.40	.001342	.000918	.000708	.000579	.000489	.010125
. 45	.001608	.001150	.000921	.000778	.000678	.019374
.50	.001883	.001398	.001154	.001000	.000892	.039637
.55	.002166	.001659	.001403	.001241	.001127	.078530
.60	.002455	.001929	.001664	.001498	.001380	.138344
.65	.002747	.002206	.001936	.001766	.001647	.220950
.70	.003040	.002489	.002215	.002044	.001924	.333566
.75	.003331	.002772	.002497	.002326	.002207	.491739
.80	.003614	.003051	.002776	.002607	.002490	.727287
.85	.003877	.003315	.003043	.002877	.002763	1.113018
.90	.004086	.003533	.003268	.003108	.002998	1.857107
.95	.004092	.003574	.003328	.003180	.003079	3.886786

		a = .006	51	CV = 9.00		
		AUTOCO	DERELATION	LAG N		
RHO	₩= 1	2	3	4	5	TOTAL
.05	.000027	.000005	.000001	.000000	.000000	.000033
.10	.000100	.000030	.000011	.000004	.000002	.000150
.15	.000208	.000084	.000040	.000021	.000012	.000383
.20	.000344	.000167	.000095	.000059	.000038	.000789
.25	.000501	.000276	.000176	.000121	.000087	.001466
.30	.000677	.000409	.000283	.000210	.000162	.002596
.35	.000866	.000562	.000414	.000325	.000265	.004567
.40	.001068	.000732	.000566	.000464	.000393	.008313
. 45	.001280	.000918	.000737	.000623	.000544	.016425
.50	.001500	.001116	.000923	.000801	.000716	.035736
•55	.001726	.001324	.001122	.000995	.000905	.075090
.60	.001956	.001540	.001331	.001200	.001108	.136040
.65	.002191	.001763	.001550	.001417	.001323	.219785
.70	.002427	.001990	.001774	.001640	.001547	.333649
.75	.002662	.002220	.002003	.001869	.001776	.493542
.80	.002893	.002447	.002231	.002098	.002008	.732039
.85	.003112	.002666	.002451	.002322	.002233	1.124028
.90	.003297	.002856	.002646	.002520	.002435	1.885754
.95	.003348	.002929	.002732	.002615	.002536	4.002704

		a005	50	cv = 10.00		
		AUTOCO	DERELATION	LAG N		
EHO	₩= 1	2	3	4	5	TOTAL
.05	.000022	.000004	.000001	.000000	.000000	.000027
.10	.000082	.000025	.000009	.000004	.000002	.000122
.15	.000170	.000069	.000033	.000017	.000010	.000312
.20	.000280	.000136	.000078	.000048	.000031	.000644
. 25	.000408	.000225	.000144	.000099	.000071	.001199
.30	.000551	.000333	.000231	.000172	.000133	.002129
.35	.000705	.000458	.000338	.000266	.000217	.003766
.40	.000870	.000597	.000462	.000379	.000322	.006932
. 45	.001042	.000748	.000602	.000510	.000446	.014080
.50	.001221	.000910	.000754	.000655	.000586	.032517
.55	.001406	.001080	.000916	.000814	.000741	.072337
.60	.001594	.001257	.001088	.000982	.000908	.134256
.65	.001786	.001439	.001267	.001160	.001085	.218882
.70	.001980	.001626	.001452	.001344	.001269	.333667
.75	.002174	.001815	.001640	.001532	.001458	.494811
.80	.002365	.002004	.001829	.001722	.001650	.735444
.85	.002549	.002187	.002013	.001909	.001838	1.131983
.90	.002711	.002351	.002181	.002080	.002012	1.906667
.95	.002781	.002436	.002276	.002181	.002117	4.089444

II. $M/E_k/1$

		K = 2	CV	= .707		
		AUTOCC	RRELATION	LAG N		
RHO	N= 1	2	3	4	5	TOTAL
.05	.000566	.000054	.000006	.000001	.000000	.000626
.10	.002051	.000368	.000073	.000016	.000004	.002513
.15	.004184	.001069	.000301	.000091	.000029	.005689
.20	.006747	.002185	.000776	.000295	.000118	.010204
.25	.009558	.003684	.001548	.000694	.000326	.016129
.30	.012470	.005501	.002630	.001338	.000711	.023560
.35	.015359	.007548	.003999	.002247	.001315	.032623
.40	.018116	.009729	.005603	.003413	.002161	.043478
. 45	.020645	.011941	.007371	.004796	.003239	.056328
.50	.022857	.014080	.009212	.006336	.004513	.071429
.55	.024665	.016037	.011024	.007945	.005919	.089102
.60	.025978	.017701	.012696	.009521	.007368	.109756
.65	.026697	.018950	.014100	.010941	.008746	.133914
.70	.026708	.019652	.015097	.012065	.009917	.162252
.75	.025871	.019651	.015524	.012728	.010716	.195652
.80	.024010	.018760	.015188	.012734	.010945	.235294
.85	.020889	.016738	.013849	.011840	.010362	.282779
.90	.016187	.013267	.011190	.009732	.008652	.340336
.95	.009449	.007903	.006781	.005988	.005397	.411162

		к == 3	ev	= .577		
		AUTOCO	DERELATION	LAG N		
EHO	N= 1	2	3	4	5	TOTAL.
.05	.000763	.000065	.000006	.000001	.000000	.000835
. 10	.002798	.000452	.000084	.000017	.000004	.003356
.15	.005777	.001339	.000353	.000101	.000030	.007614
.20	.009431	.002787	.000931	.000336	.000128	.013699
.25	.013536	.004786	.001902	.000815	.000367	.021739
.30	.017904	.007280	.003308	.001615	.000827	.031915
.35	.022371	.010181	.005148	.002788	.001580	.044465
.40	.026795	.013382	.007385	.004351	.002678	.059701
. 45	.031042	.016763	.009950	.006286	.004140	.078035
.50	.034985	.020193	.012746	.008538	.005949	.100000
.55	.038494	.023530	.015653	.011018	.008052	.126305
.60	.041423	.026617	.018525	.013603	.010352	.157895
. 65	.043604	.029272	.021187	.016136	.012711	.196056
.70	.044824	.031276	.023425	.018412	.014942	.242574
.75	.044800	.032348	.024966	.020170	.016795	.300000
.80	.043128	.032106	.025444	.021054	.017926	.372093
.85	.039203	.029993	.024332	.020562	.017850	.464630
.90	.032060	.025137	.020821	.017923	.015824	.586957
.95	.020036	.016061	.013552	.011857	.010625	.755230

		K == 4	CV	= .500		
		AUTOCC	PRELATION	LAG N		
RHO	N= 1	2	3	4	5	TOTAL
.05	.000863	.000069	.000006	.000001	.000000	.000939
.10	.003184	.000486	.000087	.000017	.000003	.003778
.15	.006615	.001452	.000369	.000102	.000030	.008582
.20	.010869	.003051	.000986	.000347	.000129	.015464
.25	.015705	.005291	.002039	.000852	.000375	.024590
.30	.020921	.008127	.003590	.001714	.000860	.036193
.35	.026341	.011480	.005657	.003002	.001672	.050585
.40	.031807	.015247	.008218	.004755	.002881	.068182
. 45	.037174	.019308	.011216	.006972	.004528	.089536
.50	.042301	.023531	.014562	.009613	.006618	.115385
.55	.047041	.027765	.018138	.012602	.009112	.146726
.60	.051229	.031840	.021795	.015820	.011927	.184932
.65	.054665	.035554	.025345	.019105	.014929	.231930
.70	.057092	.038652	.028549	.022236	.017919	.290514
.75	.058145	.040794	.031085	.024911	.020618	.364865
.80	.057277	.041486	.032494	.026697	.022614	.461538
.85	.053600	.039948	.032062	.026924	.023271	.591405
.90	.045546	.034829	.028566	.024454	.021511	.773885
.95	.030032	.023504	.019653	.017113	.015287	1.047389

		K == 5	CV	= .447		
		AUTOC	DERELATION	LAG N		
RHO	N= 1	2	3	4	5	TOTAL
.05	.000924	.000071	.000006	.000001	.000000	.001002
.10	.003420	.000503	.000087	.000017	.000003	.004032
.15	.007132	.001512	.000376	.000102	.000029	.009165
.20	.011764	.003196	.001011	.000349	.000128	.016529
. 25	.017070	.005575	.002108	.000867	.000376	.026316
.30	.022838	.008615	.003739	.001760	.000872	.038793
.35	.028889	.012245	.005937	.003111	.001713	.054324
. 40	.035060	.016368	.008692	.004972	.002982	.073394
. 45	.041201	.020871	.011959	.007357	.004736	.096659
.50	.047167	.025621	.015658	.010242	.006995	.125000
.55	.052805	.030471	.019678	.013560	.009738	.159631
.60	.057944	.035248	.023875	.017204	.012893	.202247
. 65	.062377	.039748	.028063	.021016	.016338	.255287
.70	.065824	.043705	.031997	.024778	.019880	.322368
.75	.067887	.046754	.035340	.028177	.023233	.409091
.80	.067938	.048345	.037588	.030744	.025958	.524590
.85	.064908	.047566	.037919	.031717	.027339	.684834
.90	.056767	.042716	.034818	.029705	.026070	.920455
.95	.039095	.030130	.025052	.021748	.019390	1.298561

		K == 6	CV	= .408		
		AUTOCO	RRELATION	LAG N		
RHO	N= 1	2	3	4	5	TOTAL.
.05	.000965	.000072	.000006	.000001	.000000	.001044
.10	.003580	.000514	.000088	.000016	.000003	.004202
. 15	.007483	.001549	.000379	.000102	.000029	.009554
.20	.012375	.003287	.001025	.000350	.000126	.017241
. 25	.018006	.005756	.002147	.000874	.000375	.027473
.30	.024163	.008930	.003829	.001784	.000876	.040541
.35	.030662	.012747	.006112	.003174	.001734	.056845
.40	.037339	.017115	.008995	.005104	.003040	.076923
. 45	.044044	.021924	.012444	.007601	.004863	.101504
.50	.050629	.027049	.016387	.010651	.007236	.131579
.55	.056944	.032344	.020722	.014198	.010148	.168524
.60	.062816	.037643	.025311	.018146	.013544	.214286
. 65	.068037	.042740	.029974	.022347	.017311	.271704
.70	.072324	.047372	.034470	.026587	.021266	.345070
.75	.075264	.051168	.038461	.030557	.025130	.441176
.80	.076194	.053555	.041427	.033779	.028457	.571429
.85	.073936	.053547	.042489	.035444	.030494	.756545
.90	.066134	.049207	.039940	.033994	.029787	1.038462
.95	.047222	.036004	.029820	.025835	.023004	1.516807

		K == 7	CV	= .378		
		AUTOCO	DESELATION	LAG N		
RHO	N == 1	2	3	4	5	TOTAL
.05	.000994	.000073	.000006	.000001	.000000	.001074
.10	.003694	.000520	.000088	.000016	.000003	.004323
.15	.007736	.001574	.000381	.000101	.000029	.009832
.20	.012819	.003348	.001033	.000350	.000125	.017751
.25	.018689	.005880	.002173	.000877	.000374	.028302
.30	.025133	.009150	.003889	.001799	.000878	.041796
• 35	.031966	.013100	+006230	.003214	.001746	.058659
.40	.039024	.017646	.009205	.005193	.003077	.079470
. 45	.046156	.022681	.012785	.007769	.004948	.105013
.50	.053217	.028085	.016907	.010937	.007401	.136364
.55	.060057	.033717	.021476	.014654	.010438	.175024
.60	.066507	.039414	.026362	.018829	.014012	.223140
.65	.072360	.044978	.031390	.023326	.018022	.283875
.70	.077339	.050150	.036329	.027939	.022298	.362069
.75	.081029	.054562	.040845	.032369	.026568	.465517
.80	.082754	.057637	.044419	.036138	.030395	.607595
.85	.081279	.058354	.046147	.038420	.033009	.813321
.90	.074027	.054624	.044200	.037555	.032870	1.135514
.95	.054497	.041222	.034046	.029452	.026200	1.708202

		K = 8	CV	= .354		
		AUTOCO	RRELATION	LAG N		
RHO	N= 1	2	3	4	5	TOTAL
.05	.001016	.000073	.000006	.000001	.000000	.001096
.10	.003781	.000525	.000088	.000016	.000003	.004414
.15	.007928	.001591	.000382	.000101	.000028	.010041
.20	.013155	.003392	.001039	.000349	.000124	.018135
.25	.019209	.005971	.002190	.000879	.000373	.028926
.30	.025874	.009312	.003930	.001809	.000878	.042741
.35	.032965	.013362	.006315	.003242	.001753	.060028
.40	.040320	.018042	.009358	.005256	.003103	.081395
. 45	.047787	.023251	.013037	.007890	.005009	.107672
.50	.055224	.028871	.017296	.011149	.007522	.140000
.55	.062482	.034765	.022046	.014994	.010652	.179983
.60	.069397	.040778	.027162	.019346	.014363	.229927
.65	.075767	.046715	.032481	.024075	.018564	.293257
.70	.081321	.052326	.037777	.028988	.023095	.375274
.75	.085651	.057251	.042725	.033792	.027696	.484615
.80	.088082	.060919	.046815	.038022	.031939	.636364
.85	.087353	.062297	.049137	.040849	.035059	.859388
.90	.080747	.059203	.047793	.040555	.035465	1.216738
.95	.061021	.045877	.037809	.032671	.029043	1.877415

		K = 9	CV	= .333		
			RRELATION	LAG N		
RHO	N= 1				-	TOTAL
r.ric)	Ν= 1	2	3	4	5	IUIHL
.05	.001033	.000074	.000006	.000001	.000000	.001114
.10	.003848	.000529	.000088	.000016	.000003	.004484
.15	.008078	.001604	.000382	.000100	.000028	.010204
.20	.013419	.003425	.001042	.000348	.000124	.018433
.25	.019618	.006039	.002202	.000879	.000372	.029412
.30	.026458	.009435	.003961	.001815	.000878	.043478
.35	.033756	.013564	.006378	.003262	.001758	.061097
.40	.041347	.018350	.009474	.005302	.003121	.082902
. 45	.049084	.023695	.013231	.007982	.005053	.109756
.50	.056825	.029486	.017597	.011311	.007613	.142857
.55	.064423	.035592	.022490	.015258	.010817	.183891
.60	.071720	.041859	.027793	.019751	.014637	.235294
.65	.078520	.048102	.033347	.024668	.018991	.300712
.70	.084558	.054076	.038935	.029825	.023729	.385827
.75	.089436	.059433	.044245	.034940	.028604	.500000
.80	.092490	.063613	.048776	.039561	.033200	.659794
.85	.092454	.065586	.051627	.042868	.036761	.897516
.90	.086528	.063122	.050862	.043116	.037679	1.285714
.95	.066892	.050048	.041178	.035551	.031586	2.028090

		K = 10	C	.316		
		AUTOCO	RRELATION	LAG N		
RHO	N= 1	2	3	4	5	TOTAL.
.05	.001047	.000074	.000006	.000001	.000000	.001128
.10	.003902	.000531	.000088	.000016	.000003	.004541
.15	.008199	.001614	.000382	.000100	.000028	.010334
.20	.013632	.003451	.001045	.000348	.000123	.018672
. 25	.019948	.006093	.002211	.000880	.000370	.029801
.30	.026931	.009533	.003985	.001819	.000877	.044070
.35	.034396	.013725	.006428	.003277	.001761	.061956
.40	.042182	.018595	.009565	.005338	.003134	.084112
. 45	.050140	.024051	.013384	.008054	.005088	.111434
.50	.058131	.029982	.017837	.011439	.007684	.145161
.55	.066013	.036260	.022848	.015469	.010948	.187049
.60	.073629	.042738	.028302	.020077	.014857	.239645
.65	.080789	.049234	.034051	.025148	.019336	.306777
.70	.087239	.055514	.039884	.030508	.024246	.394454
.75	.092592	.061239	.045499	.035885	.029350	.512658
.80	.096195	.065863	.050410	.040842	.034247	.679245
.85	.096796	.068371	.053730	.044572	.038197	.929593
.90	.091549	.066511	.053513	.045325	.039588	1.345018
.95	.072195	.053805	.044209	.038140	.033871	2.163116

AUTOCORRELATION LAG N RHO N= 1 2 3 4 5 TOTAL .05 .001088 .000075 .000006 .000001 .000000 .001169 .10 .004066 .000538 .000087 .000016 .000003 .004711 .15 .008565 .001642 .000382 .000099 .000027 .010725 .20 .014279 .003525 .001051 .000345 .000120 .019391 .25 .020954 .006249 .002236 .000879 .000366 .030973 .30 .028376 .009818 .004050 .001829 .000874 .045852 .35 .036363 .014198 .006569 .003317 .001768 .064546 .40 .044755 .019326 .009830 .005439 .003169 .087774 .45 .053410 .025121 .013835 .008261 .005183 .116523 .50 .062196 .031484 .018554 .011816 .007890 .152174 .55 .070982 .038302 .023925 .016097 .011334 .196702 .60 .079631 .045445 .029856 .021061 .015515 .253012 .65 .087978 .052757 .036222 .026620 .020388 .325537 .70 .095807 .060037 .042847 .032632 .025847 .421376 .75 .102791 .066997 .049475 .038873 .031703 .552632 .80 .108366 .073168 .055693 .044971 .037619 .741722 .85 .111403 .077650 .060716 .050221 .042951 1.035312 .90 .109148 .078299 .062710 .052982 .046199 1.549180							
№ 1 2 3 4 5 TOTAL .05 .001088 .000075 .000006 .000001 .000000 .001169 .10 .004066 .000538 .000087 .000016 .000003 .004711 .15 .008565 .001642 .000382 .000099 .000027 .010725 .20 .014279 .003525 .001051 .000345 .000120 .019391 .25 .020954 .006249 .002236 .000879 .000366 .030973 .30 .028376 .009818 .004050 .001829 .000874 .045852 .35 .036363 .014198 .006569 .00317 .001768 .064546 .40 .044755 .019326 .009830 .005439 .003169 .087774 .45 .053410 .025121 .013835 .008261 .005183 .116523 .50 .062196 .031484 .018554 .011816 .007890 .152174 <th></th> <th></th> <th>K = 15</th> <th>C,</th> <th>✓ = •258</th> <th></th> <th></th>			K = 15	C,	✓ = •258		
.05			AUTOCO	DRRELATION	LAG N		
.10	RHO	N= 1	2	. 3	4	5	TOTAL
.15	.05	.001088	.000075	.000006	.000001	.000000	.001169
.20 .014279 .003525 .001051 .000345 .000120 .019391 .25 .020954 .006249 .002236 .000879 .000366 .030973 .30 .028376 .009818 .004050 .001829 .000874 .045852 .35 .036363 .014198 .006569 .003317 .001768 .064546 .40 .044755 .019326 .009830 .005439 .003169 .087774 .45 .053410 .025121 .013835 .008261 .005183 .116523 .50 .062196 .031484 .018554 .011816 .007890 .152174 .55 .070982 .038302 .023925 .016097 .011334 .196702 .60 .079631 .045445 .029856 .021061 .015515 .253012 .65 .087978 .052757 .036222 .026620 .020388 .325537 .70 .095807 .066997 .049475 .038873 .031703 .552632 .80 .108366 .073168 .055693	.10	.004066	.000538	.000087	.000016	.000003	.004711
.25	.15	.008565	.001642	.000382	.000099	.000027	.010725
.30	.20	.014279	.003525	.001051	.000345	.000120	.019391
.35	.25	.020954	.006249	.002236	.000879	.000366	.030973
.40	.30	.028376	.009818	.004050	.001829	.000874	.045852
.45	.35	.036363	.014198	.006569	.003317	.001768	.064546
.50	.40	.044755	.019326	.009830	.005439	.003169	.087774
.55	. 45	.053410	.025121	.013835	.008261	.005183	.116523
.60	.50	.062196	.031484	.018554	.011816	.007890	.152174
.65 .087978 .052757 .036222 .026620 .020388 .325537 .70 .095807 .060037 .042847 .032632 .025847 .421376 .75 .102791 .066997 .049475 .038873 .031703 .552632 .80 .108366 .073168 .055693 .044971 .037619 .741722 .85 .111403 .077650 .060716 .050221 .042951 1.035312 .90 .109148 .078299 .062710 .052982 .046199 1.549180	.55	.070982	.038302	.023925	.016097	.011334	
.70	.60	.079631	.045445	.029856	.021061	.015515	.253012
.75	.65	.087978	.052757	.036222	.026620	.020388	.325537
.80 .108366 .073168 .055693 .044971 .037619 .741722 .85 .111403 .077650 .060716 .050221 .042951 1.035312 .90 .109148 .078299 .062710 .052982 .046199 1.549180	.70	.095807	.060037	.042847	.032632	.025847	.421376
.85 .111403 .077650 .060716 .050221 .042951 1.035312 .90 .109148 .078299 .062710 .052982 .046199 1.549180	.75	.102791	.066997	.049475	.038873	.031703	.552632
.85 .111403 .077650 .060716 .050221 .042951 1.035312 .90 .109148 .078299 .062710 .052982 .046199 1.549180	.80	.108366	.073168	.055693	.044971	.037619	.741722
.90 .109148 .078299 .062710 .052982 .046199 1.549180	.85	.111403	.077650	.060716			The same and the s
	.90	.109148	.078299	.062710			
104/746 1042021 2:8/124/	.95	.092437	.068064	.055694	.047946	.042521	2.671247

		K = 20	C	224		
		AUTOCC	RRELATION	LAG N		
RHO	N= 1	2	3	4	5	TOTAL
.05	.001109	.000075	.000006	.000001	.000000	.001190
.10	.004148	.000541	.000087	.000015	.000003	.004796
.15	.008749	.001654	.000382	.000098	.000027	.010921
.20	.014606	.003560	.001053	.000344	.000119	.019751
.25	.021466	.006323	.002246	.000878	.000364	.031561
.30	.029115	.009956	.004079	.001833	.000871	.046747
.35	.037372	.014430	.006634	.003334	.001769	.065851
.40	.046081	.019688	.009956	.005484	.003183	.089623
. 45	.055103	.025656	.014055	.008359	.005227	.119099
.50	.064312	.032242	.018908	.011998	.007988	.155738
• 55	.073585	.039343	.024466	.016409	.011522	.201631
.60	.082796	.046840	.030647	.021558	.015844	.259878
.65	.091800	.054592	.037342	.027375	.020924	.335247
.70	.100408	.062423	.044398	.033739	.026678	.435454
.75	.108340	.070082	.051594	.040459	.032947	.573826
.80	.115111	.077166	.058570	.047214	.039446	.775510
.85	.119731	.082885	.064644	.053391	.045615	1.094261
.90	.119691	.085303	.068161	.057514	.050108	1.669197
.95	.105947	.077527	.063305	.054438	.048245	3.005697

		v - 05	0.1			
		K = 25		/ = .200		
		AUTOCO		LAG N		
F:HO	N= 1	2	3	4	5	TOTAL
.05	.001121	.000075	.000006	.000001	.000000	.001203
.10	.004198	.000543	.000087	.000015	.000003	.004847
.15	.008861	.001662	.000382	.000097	.000026	.011038
.20	.014805	.003579	.001053	.000343	.000118	.019967
.25	.021776	.006366	.002251	.000876	.000362	.031915
.30	.029564	.010037	.004095	.001834	.000869	.047285
.35	.037987	.014567	.006672	.003343	.001769	.066636
.40	.046890	.019904	.010031	.005510	.003190	.090737
. 45	.056139	.025977	.014185	.008416	.005251	.120655
.50	.065610	.032700	.019120	.012106	.008045	.157895
.55	.075186	.039975	.024791	.016595	.011634	.204622
.60	.084751	.047691	.031126	.021857	.016041	.264059
.65	.094170	.055718	.038026	.027833	.021248	.341184
.70	.103278	.063897	.045352	.034417	.027186	.444109
.75	.111827	.072005	.052909	.041441	.033717	.586957
.80	.119393	.079685	.060379	.048622	.040592	
.85	.125105	.086244	.067159			.796680
				.055418	.047318	1.131854
.90	.126699	.089939	.071764	.060508	.052689	1.748201
.95	.115581	.084256	.068711	.059048	.052310	3.242515

		K = 50	C/	= .141		
		AUTOCC	RRELATION	LAG N		
RHO	N= 1	2	3	4	5	TOTAL
.05	.001146	.000075	.000006	.000001	.000000	.001228
.10	.004297	.000546	.000086	.000015	.000003	.004948
.15	.009085	.001675	.000381	.000096	.000026	.011274
.20	.015205	.003618	.001054	.000340	.000117	.020400
.25	.022404	.006450	.002260	.000874	.000358	.032623
.30	.030473	.010196	.004126	.001835	.000865	.048366
.35	.039235	.014838	.006744	.003358	.001768	.068214
.40	.048539	.020333	.010174	.005559	.003203	.092979
. 45	.058255	.026619	.014441	.008525	.005297	.123791
.50	.068270	.033620	.019540	.012318	.008156	.162252
.55	.078481	.041252	.025443	.016964	.011854	.210682
.60	.088789	.049422	.032094	.022458	.016435	.272559
. 65	.099093	.058026	.039419	.028763	.021904	.353315
.70	.109275	.066943	.047315	.035809	.028225	.461908
.75	.119176	.076019	.055645	.043480	.035312	.614206
.80	.128533	.085021	.064198	.051589	.043004	.841202
.85	.136806	.093510	.072587	.059789	.050985	1.212622
.90	.142543	.100367	.079857	.067226	.058478	1.924830
.95	.139546	.100942	.082107	.070466	.062372	3.827131

		K = 100)	. 100		
		AUTOCC	RRELATION	LAG N		
RHO	N= 1	2	3	4	5	TOTAL
.05	.001158	.000075	.000006	.000001	.000000	.001241
.10	.004347	.000548	.000086	.000015	.000003	.004999
.15	.009198	.001682	.000380	.000096	.000026	.011391
.20	.015406	.003636	.001054	.000339	.000116	.020616
.25	.022721	.006491	.002264	.000872	.000356	.032978
.30	.030934	.010274	.004140	.001835	.000862	.048908
.35	.039869	.014972	.006778	.003365	.001766	.069006
.40	.049378	.020546	.010244	.005581	.003208	.094106
. 45	.059336	.026939	.014566	.008578	.005319	.125371
.50	.069634	.034082	.019749	.012422	.008208	.164452
.55	.080176	.041898	.025769	.017147	.011962	.213750
.60	.090875	.050303	.032583	.022759	.016632	.276880
.65	.101649	.059208	.040128	.029235	.022235	.359513
.70	.112409	.068517	.048325	.036522	.028756	.471062
.75	.123051	.078115	.057069	.044538	.036138	.628350
.80	.133413	.087846	.066215	.053154	.044274	.864629
.85	.143185	.097445	.075521	.062149	.052963	1.256080
.90	.151531	.106254	.084419	.071011	.061738	2.023978
. 95	.154677	.111447	.090534	.077646	.068699	4.193734

		K = 100	000 DERELATION	CV = .01	.0	
RHO	N= 1	2	3	4	5	TOTAL
.05	.001171 .004397	.000076	.000006	.000001	.000000	.001253
.15	.009310	.001688	.000380	.000095	.000025	.011508
.20 .25	.015607 .023037 .031394	.003653 .006530 .010349	.001054 .002267 .004152	.000337 .000870 .001835	.000115 .000354 .000859	.020831 .033330 .049445
.35	.040503 .050220	.015103	.006811	.003371	.001764	.069793
.45	.060422 .071006	.027256	.014689	.008629 .012522	.005338	.126943
.55	.081886 .092986	.042542	.026092	.017327 .023058	.012068 .016826	.216815 .281206
.65	.104244 .115606	.060398	.040839	.029706 .037240	.022565	.365738
.75	.127026 .138464	.080251 .090755	.058516 .068288	.045611 .054759	.036975	.642710 .888642
.85	.149884 .161250	.101560	.078585 .089332	.064611	.055026 .065246	1.301333
.95	.172491	.123791	.100432	.086078	.076126	4.623463

		K = 100	0000	CV =	.001	
		AUTOCO	RRELATION	LAG N		
RHO	N= 1	2	3	4	5	TOTAL
.05	.001171	.000076	.000006	.000001	.000000	.001253
.10	.004398	.000549	.000086	.000015	.000003	.005050
.15	.009311	.001688	.000380	.000095	.000025	.011509
.20	.015609	.003653	.001054	.000337	.000115	.020833
.25	.023041	.006531	.002267	.000870	.000354	.033333
.30	.031399	.010350	.004153	.001835	.000859	.049450
.35	.040510	.015104	.006811	.003371	.001764	.069800
.40	.050229	.020758	.010312	.005602	.003213	.095238
. 45	.060433	.027259	.014690	.008629	.005339	.126959
.50	.071020	.034546	.019956	.012523	.008260	.166666
.55	.081903	.042548	.026095	.017329	.012069	.216846
.60	.093007	.051194	.033074	.023061	.016828	.281250
.65	.104270	.060410	.040846	.029711	.022569	.365800
.70	.115638	.070126	.049353	.037247	.029295	.480391
.75	.127066	.080273	.058531	.045622	.036984	.642856
.80	.138516	.090785	.068309	.054776	.045589	.888886
.85	.149953	.101602	.078617	.064637	.055048	1.301797
.90	.161351	.112667	.089384	.075128	.065282	2.131568
.95	.172686	.123926	.100540	.086170	.076208	4.628158

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E stult te					
Formulae, suitable for computation, for the lag n autocorrelation of departure intervals in the GI/M/1 queues are developed for n=1 to 6. A generating function, comparable to the one known for the M/G/1 case, is found for GI/M/1. For the Ex/M/1 and M/Ex/1 queues, expressions for arbitrary lag are given. Tables and the APL functions used to produce them are provided for Ex/M/1, H/M/1, and M/Ex/1.					